CHAPTER 1

DPP 1.1

Single Correct Answer Type

1. (c) $\cos^2\theta + \cos^2\theta + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = -\cos 2\theta$

$\Rightarrow \cos 2\theta \le 0 \Rightarrow \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

2. (b) Let the components of line segment on axes are x, y and z

So $x^2 + y^2 + z^2 = 63^2$ (1)

Now $\frac{x}{3} = \frac{y}{-2} = \frac{z}{6} = k$

$\because \quad (3k)^2 + (-2k)^2 + (6k)^2 = 63^2$

$k = \pm\frac{63}{7} = \pm 9$

$\therefore$ Components are

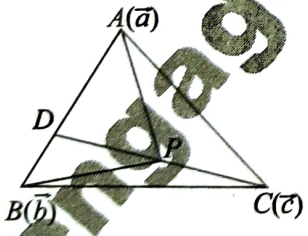
(27, –18, 54) or (–27, 18, –54)

3. (c) Let G be centroid and I be incenter.

$$|\overrightarrow{GI}| = |\overrightarrow{OI} - \overrightarrow{OG}| = \left|\frac{2\vec{a} + 3\vec{b} + 4\vec{c}}{9} - \frac{\vec{a} + \vec{b} + \vec{c}}{3}\right|$$

$$= \left|\frac{-\vec{a} + \vec{c}}{9}\right| = \frac{3}{9} = \frac{1}{3}$$

4. (c)



$$\overrightarrow{OD} = \frac{\vec{a} + 2\vec{b}}{3} = -\vec{a}, \quad (\because \vec{a} + 2\vec{b} + 3\vec{c} = \vec{0})$$

$\Rightarrow |OD| = |OC|$

Now, $\frac{\text{ar}(\Delta ACD)}{\text{ar}(\Delta BCD)} = 2,$

$\Rightarrow \text{ar}(\Delta ACD) = \frac{2\Delta}{3}$, where Δ is area of ΔABC

$\text{ar}(\Delta AOC) = \frac{1}{2}\text{ar}(\Delta ACD)$

$= \frac{1}{2} \times \frac{2\Delta}{3} = \frac{\Delta}{3}$

$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta AOC)} = 3$

5. (a) Point A is (a, b, c)

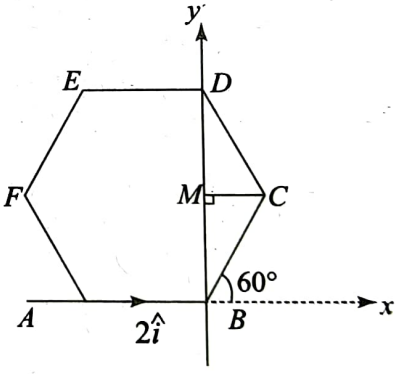
$\Rightarrow$ Points P, Q and R are $(a, b, -c)$, $(-a, b, c)$ and $(a, -b, c)$ respectively.

$\Rightarrow$ Centroid of triangle PQR is $G\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

Clearly, A, O and G are collinear.

$\Rightarrow$ Area of triangle AOG is zero.

6. (c)



$\overrightarrow{AB}$ is along the x-axis and BD is along the y-axis.

$\overrightarrow{AB} = 2\vec{i} \Rightarrow AB = BC = CD = \ldots\ldots\ldots = 2$

From the figure, $BM = BC \sin 60°$

$= 2 \sin 60°$

$= \sqrt{3}$

$\therefore \quad \overrightarrow{BD} = 2\sqrt{3}\,\vec{j}$

$\overrightarrow{BC} = BC\cos 60°\,\vec{i} + BC \sin 60°\,\vec{j}$

$= \vec{i} + \sqrt{3}\,\vec{j}$

$\therefore \quad \overrightarrow{CD} = \overrightarrow{BD} - \overrightarrow{BC} = 2\sqrt{3}\,\vec{j} - (\vec{i} + \sqrt{3}\,\vec{j})$

$= -\vec{i} + \sqrt{3}\,\vec{j}$

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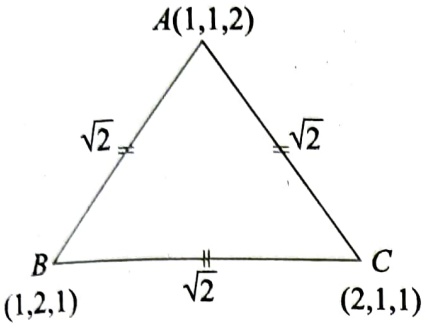
7. (c)



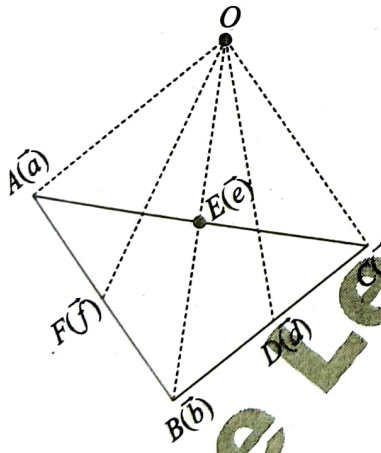
Clearly triangle formed by the given points $\hat{i}+\hat{j}+2\hat{k}$, $\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is equilateral as $AB = BC = AC = \sqrt{2}$

$\therefore$ Distance of orthocenter 'O' from the sides is equal to inradius of the triangle

$$\therefore \quad l_1 = l_2 = l_3 = \text{inradius} = r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}}{4}(\sqrt{2})^2}{\frac{3}{2}(\sqrt{2})} = \frac{1}{\sqrt{6}}$$

$$\Rightarrow \quad (l_1 + l_2 + l_3) = \frac{3}{\sqrt{6}} = \frac{\sqrt{6}}{2}$$

8. (a) Consider the point O as origin.



$$\overrightarrow{AD} + \frac{2}{3}\overrightarrow{BE} + \frac{1}{3}\overrightarrow{CF}$$

$$= (\vec{d} - \vec{a}) + \frac{2}{3}(\vec{e} - \vec{b}) + \frac{1}{3}(\vec{f} - \vec{c})$$

$$= \frac{1}{2}(\vec{c} - \vec{a})$$

$$\left(\text{Putting } \vec{d} = \frac{\vec{b}+\vec{c}}{2}, \vec{e} = \frac{\vec{a}+\vec{c}}{2}, \vec{f} = \frac{\vec{a}+\vec{b}}{2}\right)$$

$$= \frac{1}{2}(\overrightarrow{AC})$$

9. (a) We know that line joining the mid-points of opposite edges of tetrahedron are concurrent at point, which is the mean of vertices of tetrahedron. Here it is (1, –1, 2)

10. (a) Let $\vec{R}$ be the resultant of the given vectors.

Then, $\vec{R} = (2\hat{i} + 3\hat{j} - \hat{k}) + (4\hat{i} - 3\hat{j} + 2\hat{k})$

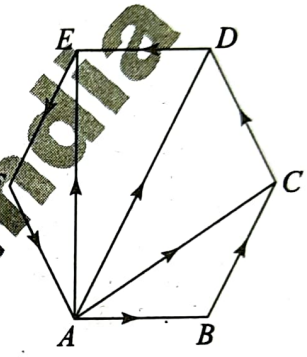
$$= 6\hat{i} + 0\hat{j} + \hat{k}$$

The required unit vector parallel to $\vec{R}$

$$= \frac{\vec{R}}{|\vec{R}|} = \frac{6\hat{i} + 0\hat{j} + \hat{k}}{\sqrt{6^2 + 0^2 + 1^2}} = \frac{6\hat{i} + 0\hat{j} + \hat{k}}{\sqrt{37}}$$

$$= \frac{1}{\sqrt{37}}(6\hat{i} + \hat{k})$$

11. (c)



$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$$

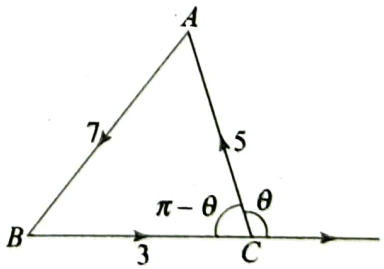
$$= \overrightarrow{ED} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{CD}$$

$$[\because \overrightarrow{AB} = \overrightarrow{ED}, \overrightarrow{AF} = \overrightarrow{CD}]$$

$$= (\overrightarrow{AC} + \overrightarrow{CD}) + (\overrightarrow{AE} + \overrightarrow{ED}) + \overrightarrow{AD}$$

$$= \overrightarrow{AD} + \overrightarrow{AD} + \overrightarrow{AD} = 3\overrightarrow{AD}$$

12. (b)



Let θ be the angle between $\vec{a}$ and $\vec{b}$

Then $\angle C = \pi - \theta$

$$\therefore \quad \cos(\pi - \theta) = \frac{3^2 + 5^2 - 7^2}{2(3)(5)}$$

$$\therefore \quad -\cos\theta = \frac{-1}{2}$$

$$\therefore \quad \theta = 60^\circ$$

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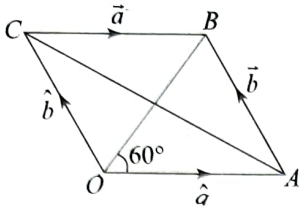
13. (b)



From the figure, $\overrightarrow{OA} = \hat{a}, \overrightarrow{OC} = \hat{b}$

$\overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{OB} = \hat{a} + \hat{b}$

Now $|\hat{a} + \hat{b}| = 1$

$\therefore$ ΔOAB is equilateral

$\therefore$ In ΔAOC, $\cos 120° = \dfrac{OA^2 + OC^2 - AC^2}{2OA \cdot OC}$

$\therefore$ $-\dfrac{1}{2} = \dfrac{1+1-AC^2}{2}$

$\therefore$ $AC = |\vec{a} - \vec{b}| = \sqrt{3}$

14. (d) $\overrightarrow{OA} = \hat{i} + 2\hat{j} - \hat{k}, \overrightarrow{OB} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{OC} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

Position vector of B w.r.t. origin at A is

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\hat{j} + 2\hat{k}$

Position vector of C w.r.t. origin at A is

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \hat{i} + \hat{j} + 3\hat{k}$

15. (b) For an equilateral triangle, centroid is the same as orthocenter

$\therefore$ $\dfrac{\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}}{3} = \vec{0}$

$\therefore$ $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \vec{0}$

Now, $\overrightarrow{AB} + 2\overrightarrow{BC} + 3\overrightarrow{CA}$

$= \overrightarrow{OB} - \overrightarrow{OA} + 2\overrightarrow{OC} - 2\overrightarrow{OB} + 3\overrightarrow{OA} - 3\overrightarrow{OC}$

$= -\overrightarrow{OB} + 2\overrightarrow{OA} - \overrightarrow{OC}$

$= -(\overrightarrow{OB} + \overrightarrow{OA} + \overrightarrow{OC}) + 3\overrightarrow{OA}$

$= 3\overrightarrow{OA}$

$= 3\vec{a}$

16. (b) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = 4\hat{i} - 5\hat{j} + 11\hat{k}$

$\therefore$ $\dfrac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \dfrac{4}{\sqrt{162}}\hat{i} - \dfrac{5}{\sqrt{162}}\hat{j} + \dfrac{11}{\sqrt{162}}\hat{k}$

$\therefore$ $\cos\gamma = \dfrac{11}{\sqrt{162}}$, where γ is the angle of $\overrightarrow{PQ}$ with z-axis.

17. (c) $\vec{a}$ and $\vec{b}$ vectors are in the same direction. $\vec{b}$ and $\vec{c}$ are in the opposite direction.

$\Rightarrow$ $\vec{a}$ and $\vec{c}$ are in opposite directions.

18. (c) $\hat{j}$ and $\hat{k}$ are unit vectors along Y- and Z- axes, then unit vector bisecting $\overrightarrow{OY}$ and $\overrightarrow{OZ}$ is $\dfrac{\hat{j} + \hat{k}}{\sqrt{2}}$

19. (b) $\vec{r} = (t^2 + 2)\hat{i} + (4t - 5)\hat{j} + (2t^2 - 6t)\hat{k}$

$\Rightarrow$ $\dfrac{d\vec{r}}{dt} = 2t\hat{i} + 4\hat{j} + (4t - 6)\hat{k}$

$\Rightarrow$ $\left(\dfrac{d\vec{r}}{dt}\right)_{t=2} = 4\hat{i} + 4\hat{j} + 2\hat{k}$

$\Rightarrow$ Unit tangent vector at $t = 2$ is $\dfrac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$

20. (d) $\overrightarrow{AC} = 2015\overrightarrow{AB}$

$\therefore$ $\vec{c} - \vec{a} = 2015(\vec{b} - \vec{a})$

$\therefore$ $\vec{c} = 2015\vec{b} - 2014\vec{a}$

21. (c) We have, $\vec{r} = \lambda\vec{c}$

Given $|\vec{r}| = |\lambda||\vec{c}|$

$\therefore$ $\dfrac{|\vec{a} + \vec{b}|}{2} = |\lambda||\vec{c}|$

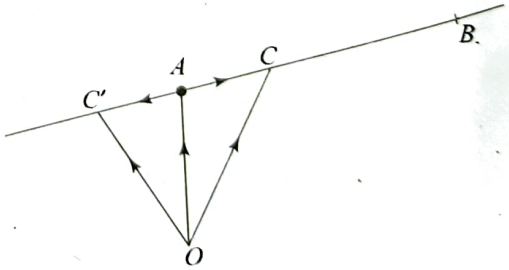
$\therefore$ $|6\hat{i} - 2\hat{j} - 4\hat{k}| = 2|\lambda||3\hat{i} - \hat{j} + 2\hat{k}|$

$\therefore$ $\sqrt{56} = 2|\lambda|\sqrt{14}$

$\therefore$ $\lambda = \pm 1$

$\therefore$ $\vec{r} = \pm\vec{c}$

22. (a)



We have $A(\hat{i} + \hat{j} - 2\hat{k})$ and $B(\hat{i} - 3\hat{j} + \hat{k})$

On line AB points C and C' are at distance 1 unit from A.

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$, where $\overrightarrow{AC}$ is unit vector in direction of $\overrightarrow{AB}$

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$\therefore \quad \overrightarrow{OC} = \overrightarrow{OA} + \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

Similarly $\overrightarrow{OC'} = \overrightarrow{OA} - \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$

Comprehension Type

For Q. 23 to 25

23. (c) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 3\vec{a} - \vec{b} - 2\vec{c}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= 9\vec{a} - 3\vec{b} - 6\vec{c}$

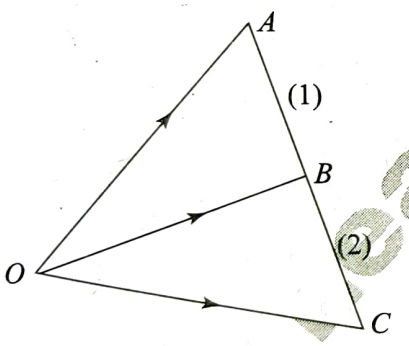
$= 3\overrightarrow{AB}$

24. (a) $2\overrightarrow{OA} - 3\overrightarrow{OB} + \overrightarrow{OC}$

$= 2(-2\vec{a} + 3\vec{b} + 5\vec{c}) - 3(\vec{a} + 2\vec{b} + 3\vec{c}) + (7\vec{a} - \vec{c})$

$= \vec{0}$

25. (d) $\because \quad 2\overrightarrow{OA} - 3\overrightarrow{OB} + \overrightarrow{OC} = \vec{O}$



$\Rightarrow \quad \overrightarrow{OB} = \dfrac{2\overrightarrow{OA} + \overrightarrow{OC}}{3}$

$\Rightarrow$ B Divides AC in 1 : 2

DPP 1.2

Single Correct Answer Type

1. (b) The vectors are linearly dependent

$$\Rightarrow \begin{vmatrix} p+1 & -3 & p \\ p & p+1 & -3 \\ -3 & p & p+1 \end{vmatrix} = 0$$

$$\Rightarrow (2p-2)\begin{vmatrix} 1 & -3 & p \\ 1 & p+1 & -3 \\ 1 & p & p+1 \end{vmatrix} = 0$$

$$\Rightarrow 2(p-1)\begin{vmatrix} 1 & -3 & p \\ 0 & p+4 & -3-p \\ 0 & p+3 & 1 \end{vmatrix} = 0$$

$\Rightarrow \quad 2(p-1)(p+4+(p+3)^2) = 0$

$\Rightarrow \quad (p-1)(p^2+7p+13) = 0$

Roots of $p^2 + 7p + 13 = 0$ are (imaginary)

$\therefore \quad p = 1$

Only integral value of p is 1.

2. (c) $\vec{a}_1 = 2\vec{b}_1 + 3\vec{b}_2 - \vec{b}_3$

$\vec{a}_2 = \vec{b}_1 - 2\vec{b}_2 + 2\vec{b}_3$

$\vec{a}_3 = -2\vec{b}_1 + \vec{b}_2 - 2\vec{b}_3$

Let $\vec{F} = \lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \lambda_3\vec{a}_3$

$= \lambda_1(2\vec{b}_1 + 3\vec{b}_2 - \vec{b}_3) + \lambda_2(\vec{b}_1 - 2\vec{b}_2 + 2\vec{b}_3)$

$+ \lambda_3(-2\vec{b}_1 + \vec{b}_2 - 2\vec{b}_3)$

$\therefore \quad \vec{F} = (2\lambda_1 + \lambda_2 - 2\lambda_3)\vec{b}_1 + (3\lambda_1 - 2\lambda_2 + \lambda_3)\vec{b}_2$

$+ (-\lambda_1 + 2\lambda_2 - 2\lambda_3)\vec{b}_3$

Also given $\vec{F} = 3\vec{b}_1 - \vec{b}_2 + 2\vec{b}_3$

$\therefore \quad (2\lambda_1 + \lambda_2 - 2\lambda_3)\ \vec{b}_1 + (3\lambda_1 - 2\lambda_2 + \lambda_3)\ \vec{b}_2$

$+ (-\lambda_1 + 2\lambda_2 - 2\lambda_3)\ \vec{b}_3 = 3\vec{b}_1 - \vec{b}_2 + 2\vec{b}_3$

Comparing coefficients of $\vec{b}_1, \vec{b}_2$ and $\vec{b}_3$

($\because$ $\vec{b}_1, \vec{b}_2$, and $\vec{b}_3$ are base vectors hence non-coplanar)

$2\lambda_1 + \lambda_2 - 2\lambda_3 = 3$

$3\lambda_1 - 2\lambda_2 + \lambda_3 = -1$

$-\lambda_1 + 2\lambda_2 - 2\lambda_3 = 2$

Solving above equations, we get

$\lambda_1 = 2$

$\lambda_2 = 5$

$\lambda_3 = 3$

$\therefore \quad \vec{F} = 2\vec{a}_1 + 5\vec{a}_2 + 3\vec{a}_3$

3. (a) Put $\Delta = 0$

$\Rightarrow \quad \lambda^7 + \lambda^3 + 2\lambda - \sin\lambda = 0$

Let $f(\lambda) = \lambda^7 + \lambda^3 + 2\lambda - \sin\lambda$

$\Rightarrow \quad f'(\lambda) = (7\lambda^6 + 3\lambda^2 + 2 - \cos\lambda) > 0 \ \forall \in R$

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$\therefore$ $f(\lambda) = 0$ has only one real solution $\lambda = 0$.

4. (a) Here $\overrightarrow{AB} = x\hat{i} - y\hat{j}$

$\overrightarrow{AC} = x\hat{i} - z\hat{k}$

$\overrightarrow{AD} = (x-1)\hat{i} - \hat{j} - \hat{k}$

As these vectors are coplanar

$$\Rightarrow \begin{vmatrix} x & -y & 0 \\ x & 0 & -z \\ x-1 & -1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$$

5. (c) $a\hat{i} + b\hat{j} + \frac{1}{2}\hat{k} = l(\hat{i} + 2\hat{j}) + m(\hat{j} - 2\hat{k})$

$\Rightarrow$ $a = l$, $b = 2l + m$ and $m = \frac{-1}{4}$

$a\hat{i} + b\hat{j} + \frac{1}{2}\hat{k}$ is unit vector

$\therefore$ $a^2 + b^2 = \frac{3}{4}$

$\Rightarrow$ $5a^2 - a - \frac{11}{16} = 0$

a_1 and a_2 are roots of above equation

$\Rightarrow$ $\frac{1}{a_1} + \frac{1}{a_2} = \frac{a_1 + a_2}{a_1 a_2} = -\frac{16}{11}$

6. (a) $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar

$\therefore$ $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

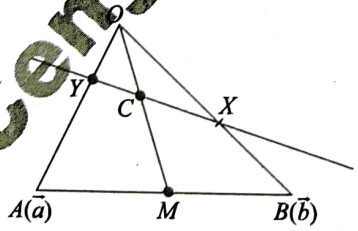
$\Rightarrow$ $c^2 - ab = 0$

For equation $ax^2 + 2cx + b = 0$

$D = 4c^2 - 4ab = 0$

So roots are real and equal.

7. (b)



$\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$

$\therefore$ $\overrightarrow{OM} = \frac{\vec{a} + \vec{b}}{2}$

$\therefore$ $\overrightarrow{OC} = \frac{\vec{a} + \vec{b}}{6}$

$\overrightarrow{OX} = \frac{2}{3}\vec{b}$

Let $\frac{OY}{YA} = \lambda$

$\therefore$ $\overrightarrow{OY} = \frac{\lambda}{\lambda + 1}\vec{a}$

Now points Y, C and X are collinear

$\therefore$ $\overrightarrow{YC} = m\overrightarrow{CX}$

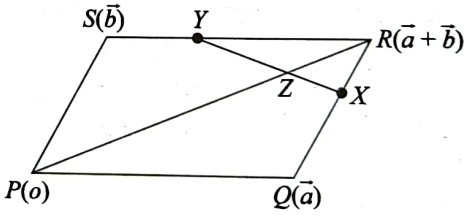
$$\frac{\vec{a} + \vec{b}}{6} - \frac{\lambda}{\lambda + 1}\vec{a} = m\frac{2\vec{b}}{3} - m\frac{\vec{a} + \vec{b}}{6}$$

Comparing coefficients of $\vec{a}$ and $\vec{b}$

$\therefore$ $\frac{1}{6} - \frac{\lambda}{\lambda + 1} = -\frac{m}{6}$ and $\frac{1}{6} = \frac{2m}{3} - \frac{m}{6}$

$\therefore$ $m = \frac{1}{3}$ and $\lambda = \frac{2}{7}$

8. (a) $\frac{4\left(\frac{5\vec{a} + 4\vec{b}}{5}\right) + \frac{\vec{a} + 5\vec{b}}{5}}{4+1} = \frac{21(\vec{a} + \vec{b})}{25} = \frac{21}{25}\overrightarrow{PR}$



P.V. of X is $\frac{4(\vec{a} + \vec{b}) + \vec{a}}{5} = \frac{5\vec{a} + 4\vec{b}}{5}$

P.V. of Y is $\frac{4\vec{b} + \vec{a} + \vec{b}}{5} = \frac{\vec{a} + 5\vec{b}}{5}$

Now, $\overrightarrow{PZ} = m\overrightarrow{PR}$

$\overrightarrow{PZ} = m(\vec{a} + \vec{b})$

Let Z divides YX in the ratio $\lambda : 1$

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P.V. of $Z = \dfrac{\lambda\overrightarrow{OX} + \overrightarrow{OY}}{\lambda+1}$

$$\therefore \quad \overrightarrow{PZ} = \frac{\lambda\left(\dfrac{5\vec{a}+4\vec{b}}{5}\right)+\dfrac{\vec{a}+5\vec{b}}{5}}{\lambda+1} = m(\vec{a}+\vec{b})$$

Comparing coefficients of $\vec{a}$ and $\vec{b}$

$$m = \frac{5\lambda+1}{5(\lambda+1)} \text{ and } m = \frac{4\lambda+5}{5(\lambda+1)}$$

$\therefore \quad \lambda = 4$

$$\therefore \quad \overrightarrow{PZ} = \frac{4\left(\dfrac{5\vec{a}+4\vec{b}}{5}\right)+\dfrac{\vec{a}+5\vec{b}}{5}}{4+1}$$

$$= \frac{21(\vec{a}+\vec{b})}{25} = \frac{21}{25}\overrightarrow{PR}$$

9. (b) Again it is given that the point P, Q and R are collinear

$\Rightarrow \quad \overrightarrow{PQ} = \lambda\overrightarrow{QR}$

$$\Rightarrow \quad 15t(4\hat{j} - 3\hat{i}) = \lambda[(1-t)(\hat{i}+\hat{j}) - 60t\hat{j}]$$
$$= \lambda[(1-t)\hat{i} + (1-61t)\hat{j}]$$

$$\Rightarrow \quad \frac{45t}{t-1} = \frac{60t}{1-61t}$$

$$\Rightarrow \quad \frac{3t}{t-1} = \frac{4t}{1-61t}$$

$\Rightarrow \quad 3(1-61t) = 4(t-1)$

$\Rightarrow \quad 3 - 183t = 4t - 4$

$\Rightarrow \quad 187t = 7$

$\Rightarrow \quad t = \dfrac{7}{187}$

Multiple Correct Answers Type

10. (b, c, d)

$$\vec{c}+\vec{a} = (\vec{b}+\vec{c}) + (\vec{a}-\vec{b})$$
$$\vec{a}+\vec{b} = (\vec{b}-\vec{c}) + (\vec{c}+\vec{a})$$
$$\vec{b}+\vec{c} = (\vec{a}+\vec{b}) + (\vec{c}-\vec{a})$$

So vectors in options (b), (c) and (d) are coplanar.

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CHAPTER 2

DPP 2.1

Single Correct Answer Type

1. (a) $\vec{\alpha}\cdot\vec{\beta}=\frac{b}{a}+\frac{4a}{b}+1\geq 5$

so $\left(\frac{10}{5+\vec{\alpha}\cdot\vec{\beta}}\right)_{\max}=1.$

2. (b) $\vec{a}=\cos\theta\hat{i}+\sin\theta\hat{j}$

$\vec{b}=-\sin\theta\hat{i}+\cos\theta\hat{j}$

and $\vec{c}=\hat{k}$

$|\vec{a}|=|\vec{b}|=|\vec{c}|$ and $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}$

$\therefore$ Required angle is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

3. (a) As $|\vec{a}+\vec{b}+\vec{c}|^2+|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$

$=3(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2)$

so, $9(GP)^2+(AB)^2+(BC)^2+(CA)^2$

$=3((PA)^2+(PB)^2+(PC)^2)$

$\Rightarrow 9\times 2^2+(a^2+b^2+c^2)=3(1^2+3^2+4^2)$

$\Rightarrow a^2+b^2+c^2=42.$

4. (c) $\vec{c}$ lies in xy-plane.

$\therefore \vec{c}=x\hat{i}+y\hat{j}$

From the given conditions $3x - y = 9$ and $x + 2y = -4$

Solving we get $\vec{c}=2\hat{i}-3\hat{j}$

$\therefore \alpha=\cos^{-1}\left(\frac{2}{\sqrt{13}}\right),\beta=\cos^{-1}\left(\frac{-3}{\sqrt{13}}\right)$

5. (b) Here $|\vec{a}|=1,|\vec{b}|=1,|\vec{c}|=1$

$\vec{a}\cdot\vec{b}=0$ and $\vec{a}\cdot\vec{c}=0$

Also, $\vec{b}\cdot\vec{c}=|\vec{b}|\cdot|\vec{c}|\cos\frac{\pi}{3}=1\cdot1\cdot\frac{1}{2}=\frac{1}{2}$

Now, $|\vec{a}+\vec{b}+\vec{c}|^2$

$=\vec{a}^2+\vec{b}^2+\vec{c}^2+2\vec{a}\cdot\vec{b}+2\vec{a}\cdot\vec{c}+2\vec{b}\cdot\vec{c}$

$=1^2+1^2+1^2+2\cdot\frac{1}{2}=4$

$\therefore |\vec{a}+\vec{b}+\vec{c}|=2$

6. (b) Let $\vec{a}=\lambda\vec{b}+\mu\vec{c}$, then $\frac{\vec{a}\cdot\vec{b}}{ab}=\frac{\vec{a}\cdot\vec{d}}{ad}$

$\Rightarrow \frac{(\lambda\vec{b}+\mu\vec{c})\cdot\vec{b}}{b}=\frac{(\lambda\vec{b}+\mu\vec{c})\cdot\vec{d}}{d}$

$\Rightarrow \frac{(\lambda\vec{b}\cdot\vec{b}+\mu\vec{c}\cdot\vec{b})}{b}=\frac{(\lambda\vec{b}\cdot\vec{d}+\mu\vec{c}\cdot\vec{d})}{d}$

$\Rightarrow \frac{5\lambda+\mu}{\sqrt{5}}=\frac{\lambda+\mu}{\sqrt{5}}$

$\Rightarrow \lambda=0$

$\therefore \vec{a}=\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

7. (b) $\overrightarrow{OG_1}\cdot\overrightarrow{BG_2}=0$

$\Rightarrow \frac{a+\vec{b}+\vec{c}}{3}\cdot\frac{a+\vec{c}-3\vec{b}}{3}=0$

$\Rightarrow a^2+c^2-3b^2+2\vec{a}\cdot\vec{c}-2\vec{b}\cdot\vec{c}-2\vec{a}\cdot\vec{b}=0$

Now, $|\vec{c}-\vec{a}|^2=b^2,|\vec{c}-\vec{b}|=a^2$ and $|\vec{a}-\vec{b}|=c^2$

$\therefore 2\vec{a}\cdot\vec{c}=a^2+c^2-b^2,\ 2\vec{b}\cdot\vec{c}=b^2+c^2-a^2,$

$2\vec{a}\cdot\vec{b}=a^2+b^2-c^2$

Putting in the above result, we get $2a^2+2c^2-6b^2=0$

$\Rightarrow \frac{a^2+c^2}{b^2}=3.$

8. (d) $p\vec{x}+q\vec{y}=\vec{a}$

$|p\vec{x}-q\vec{y}|^2=|p\vec{x}+q\vec{y}|^2-4pq\vec{x}\cdot\vec{y}\geq 0$

but $\vec{x}\cdot\vec{y}\geq\frac{|\vec{a}|^2}{4pq}$

$\Rightarrow |\vec{a}|^2=4pq\vec{x}\cdot\vec{y}$

$\Rightarrow |p\vec{x}-q\vec{y}|=0$

$\Rightarrow p\vec{x}=q\vec{y}\Rightarrow\vec{x}=\frac{\vec{a}}{2p},\vec{y}=\frac{\vec{a}}{2q}$

$\Rightarrow \frac{|\vec{x}|}{|\vec{y}|}=\frac{q}{p}$

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9. (d) $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0, |\vec{a}|=|\vec{c}|=1, |\vec{b}|=2$ and $\vec{b}\cdot\vec{c}=1$

Let $\vec{d}=x(\vec{a}+\vec{b})+y(2\vec{b}-\vec{c})$,

but $\vec{d}\cdot\vec{a}=1\Rightarrow 1=x(1+0)+0\Rightarrow x=1$

$\Rightarrow \vec{d}=\vec{a}+\vec{b}+y(2\vec{b}-\vec{c})$

$\Rightarrow |\vec{d}|^2=|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}+y^2(2\vec{b}-\vec{c})^2$
$+2y(\vec{a}+\vec{b})\cdot(2\vec{b}-\vec{c})$
$=1+4+y^2(16+1-4)+2y(8-1)$
$=13y^2+14y+5$

$\therefore |\vec{d}|_{\min}=\sqrt{\frac{4.13.5-14.14}{4.13}}=\frac{4}{\sqrt{13}}$

10. (b) We have $|\overrightarrow{OP}|^2=\left(t+\frac{1}{t}\right)^2|\vec{a}|^2+\left(t-\frac{1}{t}\right)^2|\vec{b}|^2$
$+2\left(t^2-\frac{1}{t^2}\right)|\vec{a}||\vec{b}|\cos\left(\frac{2\pi}{3}\right)$

$\therefore |\overrightarrow{OP}|^2=9\left(t+\frac{1}{t}\right)^2+4\left(t-\frac{1}{t}\right)^2$
$+2\left(t^2-\frac{1}{t^2}\right)3.2.\left(\frac{-1}{2}\right)$
$=9\left(t^2+\frac{1}{t^2}+2\right)+4\left(t^2+\frac{1}{t^2}-2\right)$
$-6\left(t^2-\frac{1}{t^2}\right)$
$=7t^2+\frac{19}{t^2}+10$

$\Rightarrow |\overrightarrow{OP}|^2\geq 2\cdot\sqrt{7t^2\cdot\frac{19}{t^2}}+10$ ($\because$ A.M. $\geq$ G.M.)

$\therefore$ minimum value of $|\overrightarrow{OP}|=\sqrt{10+2\sqrt{133}}$

11. (a) $(\vec{a}\cdot\vec{x})\vec{b}=\vec{c}+\vec{x}$ (i)

taking dot with $\vec{a}$

$(\vec{a}\cdot\vec{x})(\vec{b}\cdot\vec{a})=(\vec{c}\cdot\vec{a})+(\vec{a}\cdot\vec{x})$

$\therefore (\vec{a}\cdot\vec{x})(\vec{b}\cdot\vec{a}-1)=(\vec{c}\cdot\vec{a})$

$\therefore (\vec{a}\cdot\vec{x})=\frac{(\vec{c}\cdot\vec{a})}{(\vec{b}\cdot\vec{a}-1)}$ (ii)

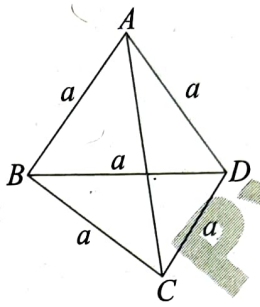
Substituting in (i)

$\left(\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}-1}\right)\vec{b}=\vec{c}+\vec{x}$

$\Rightarrow \vec{x}=\left(\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}-1}\right)\vec{b}-\vec{c}$

$=\frac{(\vec{a}\cdot\vec{c})\vec{b}-\vec{c}(\vec{a}\cdot\vec{b}-1)}{(\vec{a}\cdot\vec{b}-1)}$

12. (b)



ar $(BCD)=\frac{\sqrt{3}}{4}a^2=4\sqrt{3}$

$\Rightarrow a=4$

Vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are perpendicular

$\therefore$ ar$(\Delta PQR)=\frac{1}{2}\times 4\times 4=8$

13. (d) Let $\overrightarrow{OA}=\vec{a}, \overrightarrow{OB}=\vec{b}, \overrightarrow{OC}=\vec{c}$

Then from the given conditions

$\vec{a}\cdot\vec{a}+(\vec{b}-\vec{c})\cdot(\vec{b}-\vec{c})=\vec{b}\cdot\vec{b}+(\vec{c}-\vec{a})\cdot(\vec{c}-\vec{a})$

$\Rightarrow -2\vec{b}\cdot\vec{c}=-2\vec{c}\cdot\vec{a}$

$\Rightarrow \vec{c}\cdot(\vec{b}-\vec{a})=0$

$\Rightarrow \overrightarrow{BA}\cdot\overrightarrow{OC}=0$.

Hence $AB\perp OC$. Similarly,
$BC\perp OA$ and $CA\perp OB$

14. (a) Let θ be the required angle. Then θ will be the angle between $\vec{a}$ and $\vec{b}+\vec{c}$ ($\vec{b}+\vec{c}$ lies along the angular bisector of $\vec{a}$ and $\vec{b}$).

$\therefore \cos\theta=\frac{\vec{a}\cdot(\vec{b}+\vec{c})}{|\vec{a}||\vec{b}+\vec{c}|}$
$=\frac{2\cos\alpha}{\sqrt{2+2\cos\alpha}}=\frac{\cos\alpha}{\cos\frac{\alpha}{2}}$

$\therefore \theta=\cos^{-1}\left(\frac{\cos\alpha}{\cos\alpha/2}\right)$

15. (a) Let $\vec{r}_1=a\hat{i}+b\hat{j}+c\hat{k}$

and $\vec{r}_2=A\hat{i}+B\hat{j}+C\hat{k}$

$\Rightarrow \vec{r}_1\cdot\vec{r}_2=aA+bB+cC$

$\Rightarrow |\vec{r}_1||\vec{r}_2|=\sqrt{(a^2+b^2+c^2)(A^2+B^2+C^2)}$

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$\therefore \quad \vec{r}_1\vec{r}_2 = -|\vec{r}_1||\vec{r}_2|$

$\Rightarrow$ $\vec{r}_1$ and $\vec{r}_2$ are anti-parallel

$\Rightarrow \quad \frac{a}{A} = \frac{b}{B} = \frac{c}{C} = k$ where k is any constant

$\Rightarrow \quad \frac{aB}{bA} + \frac{bC}{cB} + \frac{cA}{aC} = 3$

DPP 2.2

Single Correct Answer Type

1. (c) $\vec{p} \times \vec{q} = (\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$= 2(\vec{a} \times \vec{b})$

$\Rightarrow \quad |\vec{p} \times \vec{q}|^2 = 4\{|\vec{b}|^2|\vec{a}|^2 - (\vec{b} \cdot \vec{a})^2\}$

$\Rightarrow \quad |\vec{p} \times \vec{q}|^2 = 4\{625 - (\vec{b} \cdot \vec{a})^2\}$

$\Rightarrow \quad |\vec{p} \times \vec{q}| = 2\{625 - (\vec{a} \cdot \vec{b})^2\}^{\frac{1}{2}}$

$\Rightarrow \quad \lambda = 625.$

2. (d) $\vec{u} \times \vec{v} = (2\hat{i} - \hat{j} + \hat{k}) \times (-3\hat{j} + 2\hat{k})$

$= \hat{i} - 4\hat{j} - 6\hat{k}$

Let $\vec{w} = a\hat{i} + b\hat{j}$

We have $a^2 + b^2 = 1$

So let $a = \cos\theta$; $b = \sin\theta$

Now $\vec{u} \times \vec{v} \cdot \vec{w} = a - 4b = \cos\theta - 4\sin\theta$

Max. value $= \sqrt{1^2 + (-4)^2} = \sqrt{17}$

3. (a) Since $\vec{a}, \vec{b}$ and $\vec{c}$ are in a plane and equally inclined to each other, then angle between any two vectors is $120°$ and $(\vec{a} \times \vec{b})$, $(\vec{b} \times \vec{c})$ and $(\vec{c} \times \vec{a})$ are parallel.

So, $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) = |\vec{a} \times \vec{b}||\vec{b} \times \vec{c}| \cdot \cos\theta$

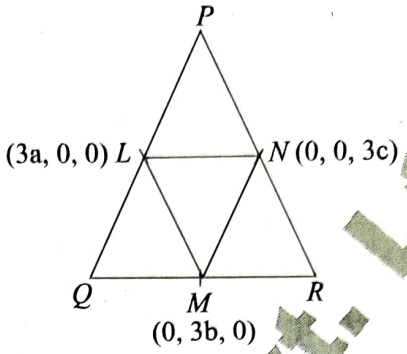
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cdot 1 = \frac{3}{4}$

Similarly, $(\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) = \frac{3}{4}$ and

$(\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = \frac{3}{4}$

$\Rightarrow \quad \text{sum} = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}.$

4. (a)



Let L, M, N be the mid-points of the sides of ΔPQR.

Area of $\Delta LMN = \frac{1}{2}|\overrightarrow{MN} \times \overrightarrow{ML}|$

$= \frac{1}{2}|(-3b\hat{j} + 3c\hat{k}) \times (3a\hat{i} - 3b\hat{j})|$

$= \frac{1}{2}|9(bc\hat{i} + ca\hat{j} + ab\hat{k})|$

$= \frac{9}{2}\sqrt{(bc)^2 + (ca)^2 + (ab)^2}$

Now, Area of $\Delta PQR = 4 \times$ (Area of ΔLMN)

5. (a) Let $\vec{b} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$

$\Rightarrow \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ \alpha & \beta & \gamma \end{vmatrix} = \hat{j} - \hat{k}$

$\Rightarrow \quad \beta - \gamma = 0, \alpha - \gamma = 1, \alpha - \beta = 1$

$\Rightarrow \quad \beta = \gamma, \alpha = 1 + \gamma, \alpha = 1 + \beta,$

$\vec{a} \cdot \vec{b} = 1$

$\Rightarrow \quad \alpha + \beta + \gamma = 1, \Rightarrow \beta + 1 + \beta + \beta = 1 \Rightarrow \beta = 0$

$\therefore \quad \alpha = 1, \gamma = 0, \quad \therefore \ \vec{b} = \hat{i}$

6. (c) $(\vec{a} - \vec{c}) \cdot (\vec{b} + \vec{c}) = 0$

$\Rightarrow \quad \vec{a} \cdot \vec{b} + \vec{c}(\vec{a} - \vec{b}) - |\vec{c}|^2 = 0$

$\Rightarrow \quad (\vec{a} - \vec{b}) \cdot \vec{c} = 1$

$\Rightarrow \quad (\vec{a} - \vec{b}) \cdot (\lambda\vec{a} + \mu\vec{b} + \omega(\vec{a} \times \vec{b})) = 1$

$\Rightarrow \quad \lambda - \mu = 1$

$\Rightarrow \quad \lambda = \mu + 1.$

Now $\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a}, \vec{b}, \vec{a} \times \vec{b}$ are mutually perpendicular.

$|\vec{c}| = 1 \quad \Rightarrow \quad \lambda^2 + \mu^2 + \omega^2 = 1$

$\Rightarrow \quad (\mu + 1)^2 + \mu^2 + \omega^2 = 1$

7. (b) $|\overrightarrow{BA}|^2 + t^2|\overrightarrow{BC}|^2 - 2\overrightarrow{BA} \cdot \overrightarrow{BC}\, t - |\overrightarrow{AC}|^2 \geq 0 \quad \forall t \in R$

Discriminant of the quadratic equation ≤ 0

$\Rightarrow 4(\overrightarrow{BA} \cdot \overrightarrow{BC})^2 - 4|\overrightarrow{BC}|^2|\overrightarrow{BA}|^2 + 4|\overrightarrow{BC}|^2|\overrightarrow{AC}|^2 \leq 0$ (1)

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Using $(\overrightarrow{BA}\cdot\overrightarrow{BC})^2-|\overrightarrow{BC}|^2|\overrightarrow{BA}|^2$

$=-|\overrightarrow{BA}\times\overrightarrow{BC}|^2$

$=-|(\overrightarrow{BC}+\overrightarrow{CA})\times\overrightarrow{BC}|^2$

$=-|\overrightarrow{CA}\times\overrightarrow{BC}|^2$ (2)

Using (2) in (1),

$|\overrightarrow{BC}|^2|\overrightarrow{AC}|^2\le|\overrightarrow{AC}\times\overrightarrow{BC}|^2$

But $|\overrightarrow{AC}\times\overrightarrow{BC}|=|\overrightarrow{AC}||\overrightarrow{BC}|\sin C$

$\Rightarrow \sin^2 C\ge 1$

$\Rightarrow \sin C=\pm1\Rightarrow\angle C=\pi/2$

8. (c) $\vec{c}\times\vec{a}=\vec{b}$

$\Rightarrow |\vec{c}\times\vec{a}|=|\vec{b}|$

$\Rightarrow |\vec{c}||\vec{a}|\sin\theta=3$

$\Rightarrow |\vec{c}|=\dfrac{3}{2\sin\theta}$

$|\vec{c}-\vec{a}|^2=|\vec{c}|^2+|\vec{a}|^2-2\vec{c}\cdot\vec{a}$

$=|\vec{c}|^2+4-2|\vec{c}|\cdot|\vec{a}|\cos\theta$

$=\dfrac{9}{4\sin^2\theta}+4-2\cdot\dfrac{3}{2\sin\theta}\cdot 2\cdot\cos\theta$

$=4+\dfrac{9}{4}\operatorname{cosec}^2\theta-6\cot\theta$

$=\dfrac{9}{4}+\left(\dfrac{3}{2}\cot\theta-2\right)^2$

$\Rightarrow |\vec{c}-\vec{a}|^2\ge\dfrac{9}{4}\Rightarrow|\vec{c}-\vec{a}|\ge\dfrac{3}{2}$

$\Rightarrow 2|\vec{c}-\vec{a}|\ge 3$

$\therefore$ Min. of $2|\vec{c}-\vec{a}|=3$

9. (d) $|\vec{a}|=1,|\vec{b}|=4,\vec{a}\cdot\vec{b}=2$

$\vec{c}=(2\vec{a}\times\vec{b})-3\vec{b}$

$\Rightarrow \vec{c}+3\vec{b}=2\vec{a}\times\vec{b}$

$\because \vec{a}\cdot\vec{b}=2$

$\Rightarrow |\vec{a}|\cdot|\vec{b}|\cos\theta=2$

$\Rightarrow \cos\theta=\dfrac{2}{|\vec{a}|\cdot|\vec{b}|}=\dfrac{2}{4}$

$\Rightarrow \cos\theta=\dfrac{1}{2}$

$\therefore \theta=\dfrac{\pi}{3}$

$\Rightarrow |\vec{c}+3\vec{b}|^2=|2\vec{a}\times\vec{b}|^2$

$\Rightarrow |\vec{c}|^2+9|\vec{b}|^2+2\vec{c}\cdot3\vec{b}=4|\vec{a}|^2|\vec{b}|^2\sin^2\theta$

$\Rightarrow |\vec{c}|^2+144+6\vec{b}\cdot\vec{c}=48$

$\Rightarrow |\vec{c}|^2+96+6(\vec{b}\cdot\vec{c})=0$ (i)

$\because \vec{c}=2\vec{a}\times\vec{b}-3\vec{b}$

taking dot product with $\vec{b}$

$\Rightarrow \vec{b}\cdot\vec{c}=0-3\times16$

$\Rightarrow \vec{b}\cdot\vec{c}=-48$

Putting value of $\vec{b}\cdot\vec{c}$ in equation (i)

$|\vec{c}|^2+96-6\times48=0$

$\Rightarrow |\vec{c}|^2=192$

Again, putting value of $|\vec{c}|$ in equation (i),

$192+96+6|\vec{b}|\cdot|\vec{c}|\cos\alpha=0$

$\Rightarrow 6\times4\times8\sqrt{3}\cos\alpha=-288$

$\Rightarrow \cos\alpha=-\dfrac{288}{6\times4\times8\sqrt{3}}=-\dfrac{3}{2\sqrt{3}}$

$\Rightarrow \cos\alpha=-\dfrac{\sqrt{3}}{2}$

$\therefore \alpha=\dfrac{5\pi}{6}$

10. (c) $|\vec{a}+\vec{b}+\vec{a}\times\vec{b}|^2+|\vec{a}+\vec{b}-\vec{a}\times\vec{b}|^2$

$=2(|\vec{a}|^2+|\vec{b}|^2+|\vec{a}\times\vec{b}|^2+2\vec{a}\cdot\vec{b})$

$=2\{1+|\vec{a}|^2+|\vec{b}|^2+|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2+2\vec{a}\cdot\vec{b}-1\}$

$=2\{(1+|\vec{a}|^2)(1+|\vec{b}|^2)-(1-\vec{a}\cdot\vec{b}|^2)\}$

11. (b) Let $\vec{r}_1=a\hat{i}+b\hat{j}+c\hat{k},\vec{r}_2=3\hat{i}+4\hat{j}+5\hat{k}$

$|\vec{r}_1\times\vec{r}_2|\le|\vec{r}_1|^2|\vec{r}_2|^2$ (1)

Now

$$\vec{r}_1\times\vec{r}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\a&b&c\\3&4&5\end{vmatrix}$$

$=\hat{i}(5b-4c)+\hat{j}(3c-5a)+\hat{k}(4a-3b)$,

So, from (1)

$(5b-4c)^2+(3c-5a)^2+(4a-3b)^2\le 50$

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Multiple Correct Answers Type

12. (a, c)

$\vec{a}\times\vec{b}=4(\vec{a}\times\vec{c})$

$\Rightarrow \vec{a}\times(\vec{b}-4\vec{c})=\vec{0}$

$\Rightarrow \vec{a}$ is parallel to $\vec{b}-4\vec{c}$

$\therefore \vec{a}=t(\vec{b}-4\vec{c})$, where t is scalar.

$\therefore |\vec{a}|^2=t^2\{|b|^2+16|c|^2-8(\vec{b}\cdot\vec{c})\}$

$\therefore 1=t^2\left\{1+16\cdot\frac{1}{16}-8.1.\frac{1}{4}.\frac{1}{2}\right\}$

$\therefore 1=t^2[1]$

$\therefore t=\pm1$

$\therefore \vec{a}=\vec{b}-4\vec{c}$ or $\vec{a}=\vec{b}+4\vec{c}$

$\Rightarrow \vec{b}=\vec{a}+4\vec{c}$ or $\vec{b}=4\vec{c}-\vec{a}$

13. (a, c, d)

$\vec{OA}\cdot\vec{OB}=-3$

$\Rightarrow 2.3\cos\theta=-3$

$\Rightarrow \cos\theta=-\frac{1}{2}\Rightarrow\theta=\frac{2\pi}{3}$

$(2(\vec{a}\times\vec{b})-2\vec{a})\cdot(2(\vec{a}\times\vec{b})-3\vec{b})$

$=4|\vec{a}|^2|\vec{b}|^2\sin^2\theta+6\vec{a}\cdot\vec{b}=4\cdot\frac{3}{4}-6\frac{1}{2}=0$

Angle C is 90°.

DPP 2.3

Single Correct Answer Type

1. (a) $\begin{vmatrix} x^2 & -1 & x \\ \lambda-1 & -2\lambda & 1 \\ 1 & -1 & 1 \end{vmatrix}>0$

$(2\lambda-1)x^2-(\lambda+1)x-(\lambda-2)<0\ \forall\ x\in R$

$2\lambda-1<0$ and $(\lambda+1)^2+4(2\lambda-1)(\lambda-2)<0$

$\Rightarrow \lambda<\frac{1}{2}$ and $9\lambda^2-18\lambda+9<0$

$\Rightarrow \lambda^2-2\lambda+1<0$ and $\lambda<\frac{1}{2}$

$\Rightarrow (\lambda-1)^2<0$ and $\lambda<\frac{1}{2}$

$\Rightarrow$ there is no value of λ.

2. (a) $\vec{x}=\vec{a}+2\vec{b}-\vec{c}$

$\vec{y}=2\vec{a}+\vec{b}+\vec{c}$

$\vec{z}=4\vec{a}-\vec{b}+5\vec{c}$

Now $\begin{vmatrix} 1 & 2 & -1 \\ 2 & 1 & 1 \\ 4 & -1 & 5 \end{vmatrix}=0$

Thus, vectors are coplanar.

$\Rightarrow [\vec{a}+2\vec{b}-\vec{c}\quad 2\vec{a}+\vec{b}+c\quad 4\vec{a}-\vec{b}+5\vec{c}]=0$

3. (d) $\vec{a}+(\vec{a}\times\vec{b})=\vec{c}$

Taking dot product with $\vec{a}$, we get $\vec{a}^2=\vec{a}\cdot\vec{c}$ and so $\vec{a}\cdot\vec{c}=1$.

Taking dot product with $\vec{c}$, we get

$\vec{a}\cdot\vec{c}+(\vec{a}\times\vec{b})\cdot\vec{c}=\vec{c}\cdot\vec{c}$

So $1+[\vec{a}\vec{b}\vec{c}]=4$

$\Rightarrow [\vec{a}\vec{b}\vec{c}]=3$

4. (a) $[\vec{a}+(\vec{a}\times\vec{b})\ \vec{b}+(\vec{a}\times\vec{b})\ \vec{a}\times\vec{b}]$

$=(\vec{a}+(\vec{a}\times\vec{b}))\cdot((\vec{b}+(\vec{a}\times\vec{b}))\times(\vec{a}\times\vec{b}))$

$=(\vec{a}+(\vec{a}\times\vec{b}))\cdot(\vec{b}\times(\vec{a}\times\vec{b}))$

$=\vec{a}\cdot\vec{b}\times(\vec{a}\times\vec{b})$

$=(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{b})$

$=1$

5. (d) $V=[\vec{a}\ \vec{b}\ \vec{c}]$

$\therefore [\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]=\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}[\vec{a}\ \vec{b}\ \vec{c}]$

$=[\vec{a}\ \vec{b}\ \vec{c}][\vec{a}\ \vec{b}\ \vec{c}][\vec{a}\ \vec{b}\ \vec{c}]=V^3$

6. (d) Translating the axes through $A(1,2,3)$.

A changes to (0, 0, 0), B changes to (8, 6, 2).

$\therefore$ Coterminous edges are of lengths 8, 6, 2.

Volume of parallelepiped = 8.6.2 = 96 cu. units

7. (d) The system of vectors is coplanar, since their sum is zero.

8. (a) $\vec{r}\cdot(3\vec{a}+5\vec{b}+2\vec{c})=0$

$\Rightarrow \vec{a}\cdot(\vec{b}\times\vec{c})\ [2\sin x+3\cos y+5]=0$

$\Rightarrow 2\sin x+3\cos y+5=0$

$\Rightarrow 2\sin x+3\cos y=-5 \qquad (\because \vec{a}\cdot(\vec{b}\times\vec{c})\neq0)$

$\Rightarrow \sin x=-1,\ \cos y=1$

$\Rightarrow \operatorname{cosec} x=-1,\ \sec y=-1.$

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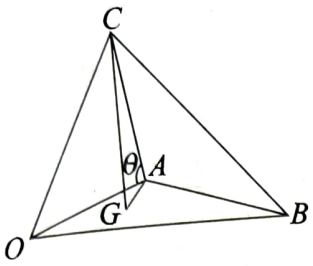
9. (c) Let $OABC$ be the tetrahedron. Let G be the centroid of the face OAB, then $GA = \frac{1}{\sqrt{3}} AC$.



Then $\cos\theta = \frac{GA}{CA} = \frac{1}{\sqrt{3}}$

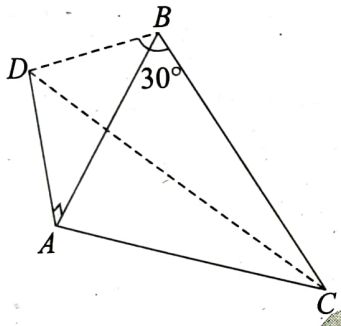
$\therefore \quad \cos^2\theta = \frac{1}{3}$

10. (a) Volume $= \frac{1}{3} AD \left(\frac{1}{2} AB \cdot BC \sin 30°\right)$

$\Rightarrow \quad 18 = \frac{1}{12} (AD \cdot AB \cdot BC)$

$\Rightarrow \quad AD \,.\, AB \,.\, BC = 216$

Now, $AB + BC + AD \geq 3\,(AD \cdot AB \cdot BC)^{1/3}$



$\Rightarrow \quad AB + BC + AD \geq 18$

Minimum value occurs when $AB = BC = AD = 6$.

Hence, $AC = \sqrt{AB^2 + BC^2 - 2AB \cdot BC \cdot \cos 30°}$

$= 6\sqrt{2-\sqrt{3}}$

Multiple Correct Answers Type

11. (a, b)

We have,

$\alpha(\vec{a} \times \vec{b}) + \beta(\vec{b} \times \vec{c}) + \gamma(\vec{c} \times \vec{a}) = 0$

Taking dot product with $\vec{c}$, we have

$\alpha[\vec{a}\,\vec{b}\,\vec{c}] = 0$

Similarly, taking dot product with b and c, we have

$\gamma[\vec{a}\,\vec{b}\,\vec{c}] = 0, \beta[\vec{a}\,\vec{b}\,\vec{c}] = 0$

Now, even if one of $\alpha, \beta, \gamma \neq 0$, then we have

$[\vec{a}\,\vec{b}\,\vec{c}] = 0$

$\Rightarrow \quad \vec{a}, \vec{b}, \vec{c}$ are coplanar

DPP 2.4

Single Correct Answer Type

1. (b) $\vec{b} = \frac{(\vec{a} \times \vec{b}) \times \vec{a} + (\vec{a} \cdot \vec{b})\vec{a}}{|\vec{a}|^2}$

$= \frac{(14\hat{i} - 29\hat{j} + \hat{k}) + (22\hat{i} + 11\hat{j} + 11\hat{k})}{6}$

$= 6\hat{i} - 3\hat{j} + 2\hat{k}$

$\Rightarrow \quad b_1 + b_2 + b_3 = 6 - 3 + 2 = 5$

2. (c) $|[\vec{a}\,\vec{b}\,\vec{d}]\vec{c} - [\vec{a}\,\vec{b}\,\vec{c}]\vec{d}|$

$= (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$

$= |\vec{a} \times \vec{b}||\vec{c} \times \vec{d}| \sin 30°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{3}{8}$

3. (d) Now $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{b} \times \vec{c}) \times (\vec{a} \times \vec{d})$

$+ (\vec{c} \times \vec{a}) \times (\vec{b} \times \vec{d})$

$= [\vec{a}\vec{b}\vec{d}]\vec{c} + [\vec{b}\,\vec{c}\,\vec{d}]\vec{a} + [\vec{c}\,\vec{a}\vec{d}]\vec{b} - 3[\vec{a}\vec{b}\vec{c}]\vec{d}$

$= [\vec{a}\vec{b}\vec{d}]\,\vec{c} + [\vec{b}\,\vec{c}\,\vec{d}]\vec{a} + [\vec{c}\,\vec{a}\vec{d}]\vec{b} - 6\vec{d}$ (i)

Now $\vec{d} = x\vec{a} + y\vec{b} + z\vec{c}$, where x, y, z are scalars

$\therefore \quad (\vec{a} \times \vec{b}) \cdot \vec{d} = z[\vec{a}\,\vec{b}\,\vec{c}] = 2z$

$\therefore \quad z = \frac{1}{2}[\vec{a}\,\vec{b}\,\vec{d}]$

Similarly, $x = \frac{1}{2}[\vec{b}\,\vec{c}\,\vec{d}]$ and

$y = \frac{1}{2}[\vec{c}\,\vec{a}\,\vec{d}]$

$\therefore$ (i) becomes $\Sigma(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$

$= 2\{z\vec{c} + x\vec{a} + y\vec{b}\} - 6\vec{d}$

$= 2\vec{d} - 6\vec{d}$

$= -4\vec{d}$

$\Rightarrow \quad \mu = 4$

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4. (b) $(\hat{p}\times\vec{q})\times\hat{p}+(\hat{p}\cdot\vec{q})\vec{q}$

$=(x^2+y^2)\vec{q}+(14-4x-6y)\hat{p}$

$\Rightarrow\quad(\hat{p}\cdot\hat{p})\vec{q}-(\vec{q}\cdot\hat{p})\hat{p}+(\hat{p}\,.\,\vec{q})\vec{q}$

$=(x^2+y^2)\vec{q}+(14-4x-6y)\hat{p}$

Since $\hat{p}$ and $\vec{q}$ are non–zero non–collinear,

we can compare coefficients of vectors $\hat{p}$ and $\vec{q}$.

$\therefore\quad 1+\hat{p}\cdot\vec{q}=x^2+y^2$ (i)

And $\hat{p}\cdot\vec{q}=4x+6y-14$ (ii)

From (i) and (ii), we get

$\Rightarrow\quad x^2+y^2-4x-6y+13=0$

$\Rightarrow\quad (x-2)^2+(y-3)^2=0$

$x=2$ and $y=3$

5. (c) $\vec{a}\times\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{c}=|\vec{c}|^2\Rightarrow[\vec{a}\,\vec{b}\,\vec{c}]=|\vec{c}|^2$

Similarly, $[\vec{b}\,\vec{c}\,\vec{a}]=|\vec{a}|^2,[\vec{c}\,\vec{a}\,\vec{b}]=|\vec{b}|^2$

$\Rightarrow\quad|\vec{a}|=|\vec{b}|=|\vec{c}|$

Also $[\vec{b}\times\vec{c}\;\vec{c}\times\vec{a}\;\vec{a}\times\vec{b}]=[\vec{a}\,\vec{b}\,\vec{c}]^2$

$\Rightarrow\quad[\vec{a}\,\vec{b}\,\vec{c}]=[\vec{a}\,\vec{b}\,\vec{c}]^2\Rightarrow[\vec{a}\,\vec{b}\,\vec{c}]=1$ as $[\vec{a}\,\vec{b}\,\vec{c}]\neq 0$

Since $\vec{a},\vec{b},\vec{c}$ are non-coplanar.

$\therefore\quad|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

$\Rightarrow\quad|\vec{a}|+|\vec{b}|+|\vec{c}|=3.$

6. (c) $\vec{F}=\alpha\hat{a}+\beta\hat{b}+\gamma\hat{c}$

$\therefore\quad\vec{F}\cdot(\hat{b}\times\hat{c})=\alpha[\hat{a}\hat{b}\hat{c}]$

$\therefore\quad\alpha=\dfrac{\hat{F}\cdot(\hat{b}\times\hat{c})}{[\hat{a}\hat{b}\hat{c}]}$

Now $\hat{b}\times\hat{c}=\hat{b}\times(\hat{a}\times\hat{b})=(\hat{b}\cdot\hat{b})\hat{a}-(\hat{b}\cdot\hat{a})\hat{b}$

$=\hat{a}-\dfrac{1}{3}\hat{b}$

$[\hat{a}\hat{b}\hat{c}]=(\hat{a}\times\hat{b})^2=|\hat{a}|^2|\hat{b}|^2-(\hat{a}\cdot\hat{b})^2$

$=1-\dfrac{1}{9}=\dfrac{8}{9}$

$\therefore\quad\alpha=\dfrac{9}{8}\left\{\hat{F}\cdot\hat{a}-\dfrac{1}{3}\hat{F}\cdot\hat{b}\right\}$

$\therefore\quad k_1=\dfrac{9}{8},\qquad k_2=\dfrac{3}{8}$

$\therefore\quad k_1+k_2=\dfrac{12}{8}=\dfrac{3}{2}$

7. (a) $(\vec{a}\times(\vec{b}\times\vec{c}))\cdot(\vec{a}\times\vec{c})=5$

$\Rightarrow\quad[(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}]\cdot(\vec{a}\times\vec{c})=5$

$\Rightarrow\quad(\vec{a}\cdot\vec{c})[\vec{b}\,\vec{a}\,\vec{c}]=5$

$\Rightarrow\quad[\vec{a}\,\vec{b}\,\vec{c}]=-10$

8. (a) $[\vec{a}\times\vec{b}\;\vec{b}\times\vec{c}\;\vec{c}\times\vec{a}]$

$=[\vec{a}\,\vec{b}\,\vec{c}]^2=|\vec{c}\cdot(\vec{a}\times\vec{b})|^2=|\vec{c}|^2|\vec{a}\times\vec{b}|^2\cos^2\theta$

The maximum value of $[\vec{a}\,\vec{b}\,\vec{c}]$ is possible only when $\vec{c}$ is parallel to $\vec{a}\times\vec{b}$.

$\Rightarrow\quad\theta=0$ or π

Hence, $|(\vec{a}\times\vec{b})\times\vec{c}|^2=|\vec{a}\times\vec{b}|^2|\vec{c}|^2\sin^2\theta=0.$

9. (b) Let θ be angle between $\vec{a}$ and plane containing $\vec{b}$ and $\vec{c}$.

$\therefore\quad 90^\circ-\theta=$ angle b/w $\vec{a}$ and $\vec{b}\times\vec{c}$.

$|\hat{a}\times(\hat{b}\times\hat{c})|^2=|(\hat{a}\cdot\hat{c})\hat{b}-(\hat{a}\cdot\hat{b})\hat{c}|^2$

$\Rightarrow\sin^2(90^\circ-\theta)\sin^2\dfrac{\pi}{4}=\left|\left(\cos\dfrac{\pi}{3}\right)\hat{b}-\left(\cos\dfrac{\pi}{6}\right)\hat{c}\right|^2$

$\Rightarrow\quad\cos^2\theta\times\dfrac{1}{2}=\dfrac{1}{4}+\dfrac{3}{4}-2\times\dfrac{1}{2}\times\dfrac{\sqrt{3}}{2}\times\dfrac{1}{\sqrt{2}}$

$=1-\dfrac{\sqrt{3}}{2\sqrt{2}}$

$\Rightarrow\quad\cos^2\theta=2-\sqrt{3/2}.$

10. (b) $\vec{a}\times(\vec{a}\times\vec{c})-\vec{b}=0$

$\Rightarrow\quad(\vec{a}\cdot\vec{c})\vec{a}-(\vec{a}\cdot\vec{a})\vec{c}-\vec{b}=0$

$\Rightarrow\quad(\vec{a}\cdot\vec{c})\vec{a}-\vec{c}=\vec{b}$

$\Rightarrow\quad|\vec{c}-(\vec{a}\cdot\vec{c})\vec{a}|^2=|\vec{b}|^2$

$\Rightarrow\quad|\vec{c}|^2+|\vec{a}\cdot\vec{c}|^2|\vec{a}|^2-2(\vec{a}\cdot\vec{c})(\vec{a}\cdot\vec{c})=|\vec{b}|^2$

If θ is the angle between $\vec{a}$ and $\vec{c}$, then

$4+(2\cos\theta)^2-2(2\cos\theta)^2=1$

$\Rightarrow\quad 4\cos^2\theta=3\quad\Rightarrow\quad\cos^2\theta=3/4$

$\Rightarrow\quad\cos\theta=\sqrt{3}/2\quad[\because\ \theta$ is acute$]$

$\Rightarrow\quad\theta=\pi/6$

11. (a) The vectors reciprocal to $\vec{a},\vec{b},\vec{c}$ are given by

$\vec{p}=\dfrac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]},\vec{q}=\dfrac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]},\vec{r}=\dfrac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ such that

$\vec{a}\cdot\vec{p}=1,\vec{a}\cdot\vec{q}=\vec{a}\cdot\vec{r}=0,$

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$\vec{b}\cdot\vec{q}=1, \vec{b}\cdot\vec{p}=\vec{b}\cdot\vec{r}=0$

$\vec{c}\cdot\vec{r}=1, \vec{c}\cdot\vec{p}=\vec{c}\cdot\vec{q}=0$

This gives $(l\vec{a}+m\vec{b}+n\vec{c})\cdot(l\vec{p}+m\vec{q}+n\vec{r})$
$=l^2+m^2+n^2$.

12. (b) $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$

$\vec{A}\times(\vec{R}\times\vec{B})=\vec{A}\times(\vec{C}\times\vec{B})$

$\Rightarrow (\vec{A}\cdot\vec{B})\vec{R}-(\vec{A}\cdot\vec{R})\vec{B}=(\vec{A}\cdot\vec{B})\vec{C}-(\vec{A}\cdot\vec{C})\vec{B}$

$\Rightarrow (-38)\vec{R}-\vec{O}=(-38)\vec{C}-19\vec{B}$

$\Rightarrow \dfrac{|\vec{B}|}{|\vec{R}-\vec{C}|}=2$

Multiple Correct Answers Type

13. **(a, b, d)**

Volume $=|[2\vec{b}\times\vec{c}\quad 3\vec{c}\times\vec{a}\quad 4\vec{a}\times\vec{b}]|=18$

$\Rightarrow 24[\vec{a}\,\vec{b}\,\vec{c}]^2=18$

$\Rightarrow |[\vec{a}\,\vec{b}\,\vec{c}]|=\dfrac{\sqrt{3}}{2}$

Now,

$$[\vec{a}\vec{b}\vec{c}]=\begin{vmatrix}(1+\sin\theta) & \cos\theta & \sin 2\theta\\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right)\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$$

Applying $R_1\to R_1+R_2+R_3$ and expanding

$|[\vec{a}\vec{b}\vec{c}]|=\sqrt{3}\,|\cos 3\theta|=\dfrac{\sqrt{3}}{2}$

$\Rightarrow \cos 3\theta=\pm\dfrac{1}{2}\Rightarrow 3\theta=\dfrac{\pi}{3},\dfrac{2\pi}{3},\dfrac{4\pi}{3}$

$\Rightarrow \theta=\dfrac{\pi}{9},\dfrac{2\pi}{9},\dfrac{4\pi}{9}$.

14. **(a, b, c, d)**

Since $\vec{a},\vec{b}$ and $\vec{a}\times\vec{b}$ are non-coplanar.

So, $\vec{r}=x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b})$

Given $\vec{r}\times\vec{b}=\vec{a}$

$\Rightarrow x\vec{a}\times\vec{b}+z\{(\vec{a}\cdot\vec{b})\vec{b}-(\vec{b}\cdot\vec{b})\vec{a}\}=\vec{a}$

$\Rightarrow -(1+z|\vec{b}|^2)\vec{a}+x\vec{a}\times\vec{b}=0$ [since $\vec{a}\cdot\vec{b}=0$]

$\therefore\quad x=0$ and $z=-\dfrac{1}{|\vec{b}|^2}$

Thus $\vec{r}=y\vec{b}-\dfrac{\vec{a}\times\vec{b}}{|\vec{b}|^2}$, where y is the parameter.

15. **(b, c, d)**

(a) $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=\vec{0}$

$\Rightarrow$ Vectors are coplanar, so do not form RHS.

(b) $(\vec{a}\times\vec{b})\times\vec{c},\vec{a}\times\vec{b},\vec{c}$ in that order form RHS.

$\Rightarrow \vec{c},(\vec{a}\times\vec{b})\times\vec{c},\vec{a}\times\vec{b}$ also form RHS as they are in same cyclic order.

(c) $\vec{a}+\vec{b}+\vec{c}=\vec{0}\Rightarrow(\vec{a}+\vec{b}+\vec{c})^2=0$

$\Rightarrow \vec{a}^2+\vec{b}^2+\vec{c}^2=-2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

Hence $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}<0$

(d) $\vec{a}+\vec{b}+\vec{c}=\vec{0}\Rightarrow\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Using this we get result.

16. **(a, b, c)**

$|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

and $\vec{a}\cdot\vec{c}=\vec{b}\cdot\vec{c}$ (i)

$\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})$
$=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}+(\vec{a}\cdot\vec{b})\vec{c}-(\vec{b}\cdot\vec{c})\vec{a}$
$=(\vec{a}\,.\,\vec{c})\,\vec{b}-(\vec{b}\,.\,\vec{c})\,\vec{a}$
$=(4+x^2)\vec{b}-(4x\cos^2\theta)\vec{a}$

$\Rightarrow \vec{a}\cdot\vec{c}=4+x^2$ and $\vec{b}\cdot\vec{c}=4x\cos^2\theta$ (ii)

From (i) and (ii), $4+x^2=4x\cos^2\theta$
$\Rightarrow 4+x^2=4x-4x\sin^2\theta$
$\Rightarrow (x-2)^2=-4x\sin^2\theta$
$\Rightarrow x=2$ or $\sin\theta=0$.

17. **(a, c)**

$(\vec{a}-\vec{b})\times[(\vec{b}+\vec{a})\times(2\vec{a}+\vec{b})]=\vec{b}+\vec{a}$

$\Rightarrow \{(\vec{a}-\vec{b})\cdot(2\vec{a}+\vec{b})\}(\vec{b}+\vec{a})$
$-\{(\vec{a}-\vec{b})\cdot(\vec{b}+\vec{a})\}(2\vec{a}+\vec{b})=\vec{b}+\vec{a}$

$\Rightarrow (2-\vec{a}\cdot\vec{b}-1)(\vec{b}+\vec{a})=\vec{b}+\vec{a}$

$\Rightarrow \vec{b}+\vec{a}=\vec{0}$ or $1-\vec{a}\cdot\vec{b}=1$

$\Rightarrow \vec{b}=-\vec{a}$ or $\vec{a}\cdot\vec{b}=0$

$\Rightarrow \theta=\pi$ or $\theta=\dfrac{\pi}{2}$

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Comprehension Type

18. (a) $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$

$\Rightarrow\quad(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$

$=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

$\Rightarrow\quad\vec{a}$ and $\vec{c}$ are parallel.

$\therefore\quad\vec{c}$ may be equal to $4\hat{i}+2\hat{j}+4\hat{k}$

19. (d) $\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&1\\4&2&4\end{vmatrix}=-6\hat{i}+6\hat{k}$

$\therefore\quad[\vec{a}\,\vec{b}\,\vec{b}\times\vec{c}]=\begin{vmatrix}2&1&2\\1&-1&1\\-6&0&6\end{vmatrix}=-36$

$\therefore\quad$ Volume = 36

20. (a) Vector along the bisectors is

$$\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}+\frac{4\hat{i}+2\hat{j}+4\hat{k}}{6}$$

$$=\frac{1}{3}((2+\sqrt{3})\hat{i}+(1-\sqrt{3})\hat{j}+(2+\sqrt{3})\hat{k})$$

$\therefore\quad$ Vector is $(2+\sqrt{3})\hat{i}+(1-\sqrt{3})\hat{j}+(2+\sqrt{3})\hat{k}$

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CHAPTER 3

DPP 3.1

Single Correct Answer Type

1. (c) Points A and B are given and area of triangle ABC is constant.

$\therefore$ Distance of vertex C from the base AB is constant.

$\therefore$ All such points lie on infinite lines parallel to AB at constant distance.

These infinite lines form cylinder in space.

2. (b) Since the three lines are mutually perpendicular,

$\therefore \quad l_1l_2 + m_1m_2 + n_1n_2 = 0$

$l_2l_3 + m_2m_3 + n_2n_3 = 0$

$l_3l_1 + m_3m_1 + n_3n_1 = 0$

Also, $l_1^2 + m_1^2 + n_1^2 = 1,\ l_2^2 + m_2^2 + n_2^2 = 1,$

$l_3^2 + m_3^2 + n_3^2 = 1$

Now, $(l_1 + l_2 + l_3)^2 + (m_1 + m_2 + m_3)^2 + (n_1 + n_2 + n_3)^2$

$= (l_1^2 + m_1^2 + n_1^2) + (l_2^2 + m_2^2 + n_2^2)$

$+ (l_3^2 + m_3^2 + n_3^2) + 2(l_1l_2 + m_1m_2 + n_1n_2)$

$+ 2(l_2l_3 + m_2m_3 + n_2n_3)$

$+ 2(l_3l_1 + m_3m_1 + n_3n_1)$

$= 3$

$\Rightarrow \quad (l_1 + l_2 + l_3)^2 + (m_1 + m_2 + m_3)^2$

$+ (n_1 + n_2 + n_3)^2 = 3$

Here, direction cosines of required line are

$$\left(\frac{l_1 + l_2 + l_3}{\sqrt{3}}, \frac{m_1 + m_2 + m_3}{\sqrt{3}}, \frac{n_1 + n_2 + n_3}{\sqrt{3}}\right)$$

3. (b) Since $\vec{OP}$ has projections $\frac{13}{5}, \frac{19}{5}$ and $\frac{26}{5}$ on the co-ordinate axes, therefore

$$\vec{OP} = \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k}$$

Suppose P divides the line segment joining $Q(2, 2, 4)$ and $R(3, 5, 6)$ in the ratio $\lambda : 1$.

Then the position vector of P is

$$\left(\frac{3\lambda + 2}{\lambda + 1}\right)\hat{i} + \left(\frac{5\lambda + 2}{\lambda + 1}\right)\hat{j} + \left(\frac{6\lambda + 4}{\lambda + 1}\right)\hat{k}$$

$$\therefore \quad \frac{13}{5}\hat{i} + \frac{19}{5}\hat{j} + \frac{26}{5}\hat{k}$$

$$= \left(\frac{3\lambda + 2}{\lambda + 1}\right)\hat{i} + \left(\frac{5\lambda + 2}{\lambda + 1}\right)\hat{j} + \left(\frac{6\lambda + 4}{\lambda + 1}\right)\hat{k}$$

$$\Rightarrow \quad \frac{3\lambda + 2}{\lambda + 1} = \frac{13}{5}, \frac{5\lambda + 2}{\lambda + 1} = \frac{19}{5}, \frac{6\lambda + 4}{\lambda + 1} = \frac{26}{5}$$

$$\Rightarrow \quad \lambda = \frac{3}{2}$$

4. (a) Let $\vec{a} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}, \vec{b} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$ and $\vec{c} = l_3\hat{i} + m_3\hat{j} + n_3\hat{k}$.

Given that $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors.

Then $p_1\hat{i} + q_1\hat{j} + r_1\hat{k} = \vec{b} \times \vec{c} = \vec{a}$

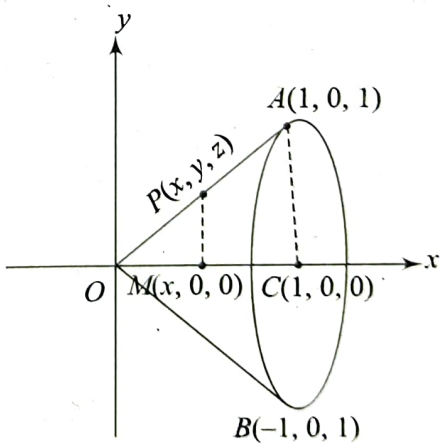
($\because \vec{b} \times \vec{c}$ parallel to $\vec{a}$ and $\vec{b} \times \vec{c}, \vec{a}$ are unit vectors)

Similarly, $p_2\hat{i} + q_2\hat{j} + r_2\hat{k} = \vec{c} \times \vec{a} = \vec{b}$

and $p_3\hat{i} + q_3\hat{j} + r_3\hat{k} = \vec{a} \times \vec{b} = \vec{c}$

These vectors are also mutually perpendicular unit vectors.

5. (b) ΔOPM and ΔOAC are similar.



$$\frac{OP}{OA} = \frac{OM}{OC}$$

$$\frac{\sqrt{x^2 + y^2 + z^2}}{\sqrt{2}} = \frac{x}{1}$$

$$\Rightarrow \quad x^2 + y^2 + z^2 = 2x^2$$

$$\Rightarrow \quad x^2 - y^2 - z^2 = 0$$

6. (c) Let point P be (p, q, r) from which three mutually perpendicular liens L_1, L_2, L_3 are drawn.

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Let line L_1 cut the x-axis at $A(a, 0, 0)$, line L_2 meet the y-axis at $B(0, b, 0)$ and it is given that line L_3 passes through the fixed point $C(0, 0, c)$.

$\therefore$ Direction ratios of L_1, L_2 and L_3 are $(p-a, q, r)$, $(p, q-b, r)$ and $(p, q, r-c)$, respectively.

Since lines are mutually perpendicular,

$$p(p-a)+q(q-b)+r^2=0 \quad (1)$$

$$p^2+q(q-b)+r(r-c)=0 \quad (2)$$

and $$p(p-a)+q^2+r(r-c)=0 \quad (3)$$

From Eq. (2) + Eq. (3) – Eq. (1), we get

$$p^2+q^2+2r(r-c)-r^2=0$$

or $$p^2+q^2+r^2-2cr=0$$

$\therefore$ Locus is $x^2+y^2+z^2-2cz=0$

7. (c) Let us suppose that A is origin.

$$ar(\Delta ABC)^2+ar(\Delta ACD)^2+ar(\Delta ABD)^2 = ar(\Delta BCD)^2$$

$$\Rightarrow \text{ar}(\Delta ACD)=\sqrt{|k_1^2+k_2^2-k_3^2|}$$

8. (c) Eliminating n we have $(l^2+m^2)-(l+m)^2=0$

$\therefore \quad 2lm=0$

When $l=0$ then $m+n=0$

$$\therefore \quad \frac{l}{0}=\frac{m}{1}=\frac{n}{-1} \quad (1)$$

When $m=0$ then $l+n=0$

$$\therefore \quad \frac{l}{1}=\frac{m}{0}=\frac{n}{-1} \quad (2)$$

$\therefore$ D.R.'s are 0, 1, –1 and 1, 0, –1

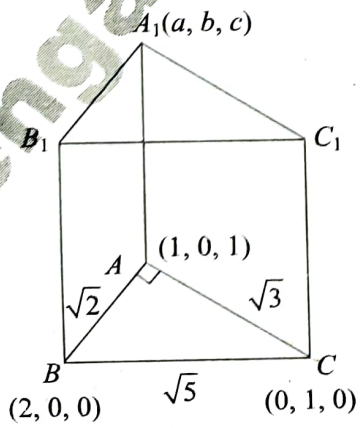
$$\therefore \quad \cos\theta=\frac{0\cdot1+1\cdot0+1}{\sqrt{1+1}\sqrt{1+1}}=\frac{1}{2}$$

$$\therefore \quad \theta=\frac{\pi}{3}$$

Multiple Correct Answers Type

9. (b, d)

Volume = Area of base × height



$$3=\frac{1}{2}\times\sqrt{2}\times\sqrt{3}\times h$$

$$\Rightarrow \quad h=\sqrt{6}$$

$$(A_1A)^2=h^2=6$$

$$\overrightarrow{A_1A}\cdot\overrightarrow{AB}=0$$

$$\overrightarrow{A_1A}\cdot\overrightarrow{AC}=0$$

$$\overrightarrow{AA_1}\cdot\overrightarrow{BC}=0$$

Solving we get position vector of A_1 as $(0, -2, 0)$ or $(2, 2, 2)$.

10. (a, c)

DC's of AB are $\frac{x_2-x_1}{AB}, \frac{y_2-y_1}{AB}, \frac{z_2-z_1}{AB}$

DC's of L are

$$\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$$

$$\cos\theta=\frac{(x_2-x_1)l+(y_2-y_1)m+(z_2-z_1)n}{AB\sqrt{l^2+m^2+n^2}}$$

Also, $PQ=AB\cos\theta$

$$=\frac{(x_2-x_1)l+(y_2-y_1)m+(z_2-z_1)n}{\sqrt{l^2+m^2+n^2}}$$

11. (a, b, c, d)

$$l+m+n=0, 4l=\frac{2mn}{m+n}$$

$$\Rightarrow \quad 2lm-mn+2ln=0$$

By eliminating n, we get

$$2\left(\frac{l}{m}\right)^2-\frac{l}{m}-1=0$$

$$\therefore \quad \frac{l_1}{m_1}=1, \frac{l_2}{m_2}=-\frac{1}{2}$$

On solving, we get

$$(l_1, m_1, n_1)=\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$$

and $$(l_2, m_2, n_2)=\left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$\frac{l_1}{l_2}+\frac{m_1}{m_2}+\frac{n_1}{n_2}=1-\frac{1}{2}-2=\frac{-3}{2}$$

$$l_1l_2+m_1m_2+n_1n_2=\frac{1}{6}-\frac{2}{6}-\frac{2}{6}=\frac{-1}{2}$$

$$l_1m_1n_1+l_2m_2n_2=\frac{-2}{6\sqrt{6}}-\frac{2}{6\sqrt{6}}$$

$$=\frac{-2}{3\sqrt{6}}=\frac{-2\sqrt{6}}{18}=\frac{-\sqrt{6}}{9}$$

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$$\therefore (l_1+l_2)(m_1+m_2)(n_1+n_2)=\frac{2}{\sqrt{6}}\times\frac{-1}{\sqrt{6}}\times\frac{-1}{\sqrt{6}}$$

$$=\frac{2}{6\sqrt{6}}=\frac{\sqrt{6}}{18}$$

DPP 3.2

Single Correct Answer Type

1. (d) Let the required ratio be $k:1$. Then the coordinates of the point which divides the line joining the points $(-2, 4, 7)$ and $(3, -5, 8)$ in this ratio are given by $\left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1}\right)$

As this point lies on the plane $2x - 1 = 0$, we have

$$\frac{3k-2}{k+1}=\frac{1}{2}\Rightarrow k=1$$

and thus the required ratio as 1 : 1.

2. (d) Line $(\vec{r}-(\hat{i}+\hat{j}+\hat{k}))\times((1-p)\hat{i}+3\hat{j}-2\hat{k})=0$ is passing through the point $A(1, 1, 1)$ and parallel to the vector $\vec{n_1}=(1-p)\hat{i}+3\hat{j}-2\hat{k}$.

Line $(\vec{r}-(3\hat{i}+\hat{j}-5\hat{k}))\times((3-p)\hat{i}+4\hat{j}-8\hat{k})=0$ is passing through the point $B(3, 1, -5)$ and parallel to the vector $\vec{n_2}=(3-p)\hat{i}+4\hat{j}-8\hat{k}$.

Since lines are coplanar, vectors $\vec{n_1}, \vec{n_2}$ and $\overrightarrow{AB}$ are coplanar.

$$\therefore \begin{vmatrix} 2 & 0 & -6 \\ (1-p) & 3 & -2 \\ (3-p) & 4 & -8 \end{vmatrix}=0$$

$$\Rightarrow p=\frac{1}{3}$$

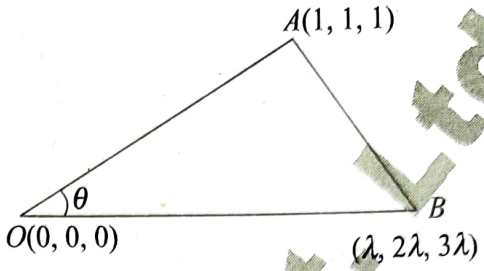
3. (c) Here P and Q lie on the same side of XY plane.

Image $P(1, 2, 3)$ on the XY plane is $P'(1, 2, -3)$.

Reflected ray is $P'Q: \frac{x-3}{3-1}=\frac{y-2}{2-2}=\frac{z-5}{5-(-3)}$

$$\Rightarrow \frac{x-3}{1}=\frac{y-2}{0}=\frac{z-5}{4}$$

4. (c) $A(1, 1, 1)$ lies on $x = y = z$ Any point on second line is $(\lambda, 2\lambda, 3\lambda)$.



$$\cos\theta=\frac{6}{\sqrt{42}},\ \therefore \sin\theta=\frac{\sqrt{6}}{\sqrt{42}}$$

Area of $\Delta OAB=\frac{1}{2}(OA)(OB)\sin\theta=\sqrt{6}$

$$\Rightarrow \frac{1}{2}(\sqrt{3})(\sqrt{14}\lambda)\frac{\sqrt{6}}{\sqrt{42}}=\sqrt{6}\Rightarrow\lambda=2$$

$\therefore$ $B = (2, 4, 6)$

5. (c) $L_1: \frac{x-0}{1}=\frac{y+1}{1}=\frac{z-0}{1}=\lambda$

$L_2: \frac{x+1}{2}=\frac{y-0}{1}=\frac{z-0}{1}=\mu$

Hence any point on L_1 and L_2 can be $(\lambda, \lambda-1, \lambda)$ and $(2\mu-1, \mu, \mu)$, respectively.

According to the question,

$$\frac{2\mu-1-\lambda}{2}=\frac{\mu-\lambda+1}{1}=\frac{\mu-\lambda}{2}$$

On solving, we get $\mu = 1$ and $\lambda = 3$

$\therefore$ $A = (3, 2, 3)$ and $B = (1, 1, 1)$

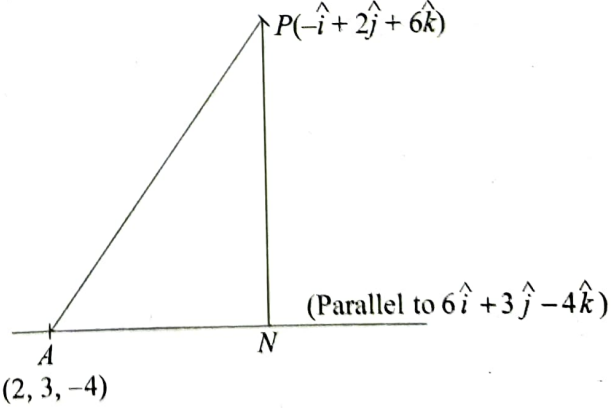
$\therefore$ $AB = 3$

6. (b) Any point on the given line is $(k+1, 2k+1, k+1)$.

but $x = 0 \Rightarrow A \equiv (0, -1, 0)$

and $y = 0 \Rightarrow B \equiv (1/2, 0, 1/2)$

7. (b)



From the figure,

$$\overrightarrow{AP}=-3\hat{i}-\hat{j}+10\hat{k}$$

$$|\overrightarrow{AP}|=\sqrt{110}$$

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$\overrightarrow{PN}$ is perpendicular to the line.

$\overrightarrow{AN}$ = The projection of $\overrightarrow{AP}$ on the line i.e., on $6\hat{i}+3\hat{j}-4\hat{k}$

$$=\left|\frac{\overrightarrow{AP}\cdot(6\hat{i}+3\hat{j}-4\hat{k})}{(6\hat{i}+3\hat{j}-4\hat{k})}\right|$$

$$=\left|\frac{-18-3-40}{\sqrt{61}}\right|=\sqrt{61}$$

$\therefore \quad PN^2 = AP^2 - AN^2 = 110 - 61 = 49$

$\therefore \quad PN = 7$

8. (d) Since point of intersection of the given lines is (0, 0, 0), it must lie on the angle bisector.

so, $\frac{0-2}{-1}=\frac{0+2}{1}=\frac{0+k}{4}$

$\Rightarrow \quad k = 8$

9. (d) Any point on the circle $x^2 + y^2 = 1$ is $P(\cos\theta, \sin\theta, 0)$

Distance of P from $O(0, 0, 0)$ which lies on the line $x = y = z$ is $OP = 1$.

Projection of OP on line having direction ratios 1, 1, 1 is

$$d=\frac{\cos\theta+\sin\theta}{\sqrt{3}}$$

$$\therefore \quad PN=\sqrt{1-\left(\frac{\cos\theta+\sin\theta}{\sqrt{3}}\right)^2}$$

$$=\frac{\sqrt{2-\sin 2\theta}}{\sqrt{3}}$$

$$\therefore \quad PN_{\min}=\sqrt{\frac{2-1}{3}}=\sqrt{\frac{1}{3}}$$

Multiple Correct Answers Type

10. (a, b)

Line through $8\hat{i}+10\hat{j}-8\hat{k}$ and parallel to $2\hat{i}+\hat{j}+2\hat{k}$ is

$$\vec{r}=(8\hat{i}+10\hat{j}-8\hat{k})+t(2\hat{i}+\hat{j}+2\hat{k})$$

$$\therefore \quad \vec{r}-(8\hat{i}+10\hat{j}-8\hat{k})=t(2\hat{i}+\hat{j}+2\hat{k})$$

$$\therefore \quad |\vec{r}-(8\hat{i}+10\hat{j}-8\hat{k})|=|t||2\hat{i}+\hat{j}+2\hat{k}|$$

$$\therefore \quad |\vec{r}-(8\hat{i}+10\hat{j}-8\hat{k})|=3|t|$$

$$\therefore \quad 12=\pm t\cdot 3\Rightarrow t=\pm 4$$

$$\vec{r}-(8\hat{i}+10\hat{j}-8\hat{k})=\pm 4(2\hat{i}+\hat{j}+2\hat{k})$$

$$\vec{r}=16\hat{i}+14\hat{j} \text{ or } \vec{r}=6\hat{j}-16\hat{k}$$

11. (a, c)

Let the direction ratios of the first and third lines be $(0, m_1, n_1)$ and (l_2, m_2, n_2).

Direction ratios of second line are 1, 2, –1.

Since lines are perpendicular,

$$2m_1 - n_1 = 0,\ l_2 + 2m_2 - n_2 = 0$$

$$\text{and } m_1m_2 + n_1n_2 = 0$$

On solving, we get

$$\Rightarrow \quad \frac{l_1}{0}=\frac{m_1}{1}=\frac{n_1}{2} \text{ and } \frac{l_2}{5}=\frac{m_2}{-2}=\frac{n_2}{1}.$$

Hence, equations of the required line are given by

$$\frac{x-1}{5}=\frac{y-2}{-2}=\frac{z+1}{1}=\lambda.$$

For $\lambda = 1$ and $\lambda = -1$, we get (a) and (c).

12. (a, b, c, d)

L_1: $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{2}$ passes through the point $A(1, -1, 0)$ and it is parallel to vector $\overrightarrow{n_1}=\hat{i}+\hat{j}+2\hat{k}$.

L_2: $\frac{x}{2}=\frac{y-1}{-2}=\frac{z-1}{\lambda}$ passes through the point $B(0, 1, 1)$ and it is parallel to vector $\overrightarrow{n_2}=2\hat{i}-2\hat{j}+\lambda\hat{k}$.

(a) If lines are parallel then $\overrightarrow{n_1}$ is parallel to $\overrightarrow{n_2}$

$$\therefore \quad \frac{2}{1}=\frac{-2}{-1}=\frac{\lambda}{2}\Rightarrow\lambda=4$$

(b) If lines are perpendicular then $\overrightarrow{n_1}\cdot\overrightarrow{n_2}=0$

$$\therefore 2+2+2\lambda=0\Rightarrow\lambda=-2$$

(c) Clearly, for $\lambda = 4$, lines are parallel and hence they are coplanar.

Coplanar for $\lambda = 4$

(d) For $\lambda = 5$, $\overrightarrow{AB}\cdot(\overrightarrow{n_1}\times\overrightarrow{n_2})\neq 0$

Hence, lines are skew.

Matching Column Type

13. (c) Lines in (p) are intersecting

$$\text{as } \begin{vmatrix} 1-1 & 2-3 & 3-5 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}=0$$

Clearly, lines in (q) have same direction ratios.

Also, points on line are $(2a + 1, 3a + 2, 4a + 3)$ and $(2b + 3, 3b + 5, 4b + 7)$

For $a = 0$ and $b = -1$, we get common point (1, 2, 3).

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So these lines are coincident.
Lines in (r) are parallel, but not coincident as points $(5a+2, 4a-3, 5-2a)$ and $(5b+7, 4b+1, 2-2b)$ do not match for any values of a and b.
Lines in (s) are skew lines as

$$\begin{vmatrix} 3-3 & -2-2 & 4-7 \\ 2 & 3 & 5 \\ 3 & 2 & 5 \end{vmatrix} \neq 0.$$

DPP 3.3

Single Correct Answer Type

1. (d) Required plane is perpendicular to the line of intersection of given two planes.
Vector along line of intersection of planes is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 3 & -4 & 1 \end{vmatrix} = 6\hat{i} + 2\hat{j} - 10\hat{k}$$

$\therefore$ Required plane is $3x + y - 5z = 0$

2. (c) Let plane make angle θ with z-axis.

$\therefore \quad \cos^2 45° + \cos^2 60° + \cos^2\theta = 1$

$\therefore \quad \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 60°$

So, equation of plane is

$$\frac{(x-\sqrt{2})}{\sqrt{2}} + \frac{(y+1)}{2} + \frac{z-1}{2} = 0$$

$$\Rightarrow \quad \frac{x}{\sqrt{2}} + \frac{y}{2} + \frac{z}{2} = 1$$

$$\Rightarrow \quad \sqrt{2}x + y + z = 2$$

3. (c) The perpendicular distance of the origin (0, 0, 0) from the plane $x + y + z = p$ is

$$\left|\frac{-p}{\sqrt{1+1+1}}\right| = \frac{|p|}{\sqrt{3}}$$

If the coordinates of P are (x, y, z) then we must have

$$\left|\frac{x+y+z-p}{\sqrt{3}}\right| = \frac{|p|}{\sqrt{3}}$$

$\Rightarrow \quad |x + y + z - p| = |p|$
Which is satisfied by (c).

4. (d) Any point on the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda$ is

$(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$
This lies on $x - y + z = 5$.
$\Rightarrow \quad 3\lambda + 2 - 4\lambda + 1 + 12\lambda + 2 = 5 \Rightarrow \lambda = 0$
So, point is $(2, -1, 2)$.
Its distance from $(-1, -5, -10)$

$$= \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$$

$$= \sqrt{9 + 16 + 144} = 13$$

5. (c) $D = \begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow k(k-4) - 4(0) + 1(8 - 2k) = 0$
$\Rightarrow k^2 - 6k + 8 = 0 \Rightarrow k = 2, 4$

6. (a) Line L is $\frac{x-1}{1} = \frac{y-2}{2\sqrt{2}} = \frac{z}{0}$

This line L makes an angle of 45° with the plane $\sqrt{2}x + y - z = 1$
$\therefore$ Required distance PQ is perpendicular distance of plane from P

i.e., $PQ = \frac{|\sqrt{2} + 7 - \sqrt{2} - 1|}{\sqrt{2+1+1}} = 3$

7. (b) Lines

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}, \quad \frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$$

must be coplanar.

$$\Rightarrow \quad \begin{vmatrix} 2 & 3 & \lambda \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

$\Rightarrow \quad \lambda = 4$

$$\therefore \text{ Equation of plane is } \begin{vmatrix} x-3 & y-2 & z-1 \\ 2 & 3 & 4 \\ 3 & 2 & 3 \end{vmatrix} = 0$$

or $\quad x + 6y - 5z = 10 \qquad (1)$
Equation of given plane is $4x + y + 2z = 0 \qquad (2)$
Angle between the planes i.e., 'θ' is given by

$$\cos\theta = \frac{4(1) + 6(1) - 5(2)}{\sqrt{1+36+15}\sqrt{16+1+4}} = 0$$

$\Rightarrow \quad \theta = 90°$

8. (a) Equation of line parallel to given line and through $(1, -2, 3)$ is $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} \qquad (1)$

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Any point on (1) is $P(2r+1, 3r-2, -6r+3)$

Putting 'P' in the given plane, $r = \frac{1}{7}$.

$\therefore$ Line meets the given plane at $P\left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$

$\therefore$ Required distance

$$= \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(\frac{11}{7}-2\right)^2 + \left(\frac{15}{7}-3\right)^2} = 1 \text{ unit}$$

9. (c) $2x^2 - 2y^2 + 4z^2 + 6xz + 2yz + 3xy = 0$

or $2x^2 + x(6z+3y) - 2y^2 + 4z^2 + 2yz = 0$

$$\therefore \quad x = \frac{-(6z+3y) \pm \sqrt{\begin{matrix}36z^2+9y^2+36yz \\ -8(-2y^2+4z^2+2yz)\end{matrix}}}{4}$$

$$\therefore \quad x = \frac{-(6z+3y) \pm \sqrt{(2z+5y)^2}}{4}$$

$$\therefore \quad x = \frac{-(6z+3y) \pm (2z+5y)}{4}$$

or $2x - y + 2z = 0, \; x + 2y + 2z = 0$

$\therefore$ Angle between planes

$$\theta = \cos^{-1}\frac{(2)(1)+(-1)(2)+(2)(2)}{\sqrt{(2)^2+(-1)^2+(2)^2}\sqrt{(1)^2+(2)^2+(2)^2}}$$

$$= \cos^{-1}\left(\frac{4}{9}\right)$$

10. (c) We have

$$\vec{r} = (1+\lambda-\mu)\hat{i} + (2-\lambda)\hat{j} + (3-2\lambda+2\mu)\hat{k}$$

$$\Rightarrow \quad \vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(\hat{i}-\hat{j}-2\hat{k}) + \mu(-\hat{i}+2\hat{k})$$

This is a plane passing through $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$ and parallel to the vectors $\vec{b} = \hat{i}-\hat{j}-2\hat{k}$ and $\vec{c} = -\hat{i}+2\hat{k}$.

Therefore, it is perpendicular to the vector $\vec{n} = \vec{b} \times \vec{c} = -2\hat{i} - \hat{k}$.

Hence, its vector equation is

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

$$\Rightarrow \quad \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$$

$$\Rightarrow \quad \vec{r} \cdot (-2\hat{i} - \hat{k}) = -2 - 3$$

$$\Rightarrow \quad \vec{r} \cdot (2\hat{i} + \hat{k}) = 5$$

So, the Cartesian equation is

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + \hat{k}) = 5$$

or $2x + z = 5$

11. (d) $xy + yz = 0$

$\therefore \quad y(x+z) = 0$

$\therefore \quad y = 0$ or $x + z = 0$

This is a pair of perpendicular planes.

12. (d) Any point on the line $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{1}$ is

$B(2\lambda, 1+3\lambda, 2+\lambda)$

Now, AB is parallel to the plane $x + y + z = 2$, where $A \equiv (1, 0, 3)$.

$\therefore \quad (2\lambda - 1) + (1 + 3\lambda) + (\lambda - 1) = 0$

$\Rightarrow \quad \lambda = \frac{1}{6}$

$\therefore$ Point $B \equiv \left(\frac{1}{3}, \frac{3}{2}, \frac{13}{6}\right)$

Direction ratio's of AB are

$$\frac{1}{3}-1, \frac{3}{2}-0, \frac{13}{6}-3 \equiv \frac{-2}{3}, \frac{3}{2}, \frac{-5}{6}$$

$\therefore$ Its equation is

$$\frac{x-(1/3)}{-(2/3)} = \frac{y-(3/2)}{(3/2)} = \frac{z-(13/6)}{-(5/6)}$$

or $\frac{3x-1}{2} = \frac{2y-3}{-3} = \frac{6z-13}{5}$

13. (d) We have $Q(-1, 0, 1)$ and $R(1, 0, -1)$.

$\therefore \quad QR = 2\sqrt{2}$

Now, mid point of QR is $M(0, 0, 0)$.

Triangle PQR is equilateral

$\therefore \; PM = MQ \times \tan 60° = \sqrt{2}\sqrt{3} = \sqrt{6}$

Thus, P can lie on the plane whose distance from point M is less than or equal to $\sqrt{6}$.

Hence P can lie on the plane $x + y + z + 3\sqrt{2} = 0$.

14. (d) The plane is $x + 3y + z - 4 + \lambda(2x - y) = 0$

This always passes through the line of intersection of the planes $x + 3y + z - 4 = 0$ and $2x - y = 0$.

Now, $2x - y = 0 \Rightarrow \quad \frac{x}{1} = \frac{y}{2}$

$\therefore \quad x + 3y + z - 4 = 0$

$\Rightarrow \quad x + 3 \cdot 2x + z - 4 = 0$

$\Rightarrow \quad 7x + z - 4 = 0$

$\Rightarrow \quad 7x = -(z - 4)$

$\Rightarrow \quad \frac{x}{1} = \frac{z-4}{-7}$

$\therefore$ line is $\frac{x}{1} = \frac{y}{2} = \frac{z-4}{-7}$

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Multiple Correct Answers Type

15. (b, c)
Required line is parallel to the vector

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$$
$$[\vec{a}\,\vec{c}\,\vec{d}]\vec{b}-[\vec{b}\,\vec{c}\,\vec{d}]\vec{a}$$
$$=4\vec{b}-4\vec{a}=-8\hat{i}$$

which is perpendicular to y-axis and z-axis.

16. (b, c)

$$E_1:\ \vec{r}\times(2\hat{i}-\hat{j}+3\hat{k})=3\hat{i}+\hat{k}$$
$$\Rightarrow\ \hat{i}(3y+z)-\hat{j}(3x-2z)+\hat{k}(-x-2y)=3\hat{i}+\hat{k}$$
$$\therefore\ 3y+z=3,\ 3x-2z=0,\ -x-2y=1$$

From first two equations, $3x-2(3-3y)=0$
$\Rightarrow\ 3x+6y=6\Rightarrow x+2y=2$
Now, $x+2y=-1$, $x+2y=2$ are parallel planes.

$$E_2:\ \vec{r}\times(\hat{i}+2\hat{j}-3\hat{k})=2\hat{i}-\hat{j}$$
$$\Rightarrow \hat{i}(-3y-2z)-\hat{j}(-3x-z)+\hat{k}(2x-y)=2\hat{i}-\hat{j}$$
$$\therefore\ -3y-2z=2,\ 3x+z=-1,\ 2x-y=0$$

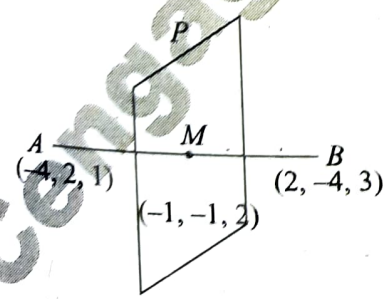
i.e., $-6x-2z=2$, $3x+z=-1$
$\therefore$ Straight line, $2x-y=0$, $3x+z=-1$

17. (b, c)
For $A(1, 2, 3)$, $2x-y-3z-5=2-1-3-5<0$
For $B(2, 1, -3)$, $2x-y-3z-5=4-1+9-5>0$
For $C(1, -2, -2)$, $2x-y-3z-5$
$=2+2+6-5>0$
For $D(-3, 1, 2)$, $2x-y-3z-5$
$=-6-1-6-5<0$

$\therefore$ A, D are on one side of the plane and B, C are on the other side
$\therefore$ The line segments AB, AC, BD and CD intersect the plane.

Comprehension Type

18. (c)



Plane P will contain the point $M(-1, -1, 2)$ and dr's of its normal will be proportional to $(3, -3, 1)$.

Thus, equation of the plane P is
$3(x+1)-3(y+1)+1(z-2)=0$
$\therefore\ P: 3x-3y+z-2=0$ (1)
Also, $Q: x-y+cz-1=0$ (2)
If P is parallel to Q then

$$\frac{3}{1}=\frac{1}{c}\ \Rightarrow\ c=\frac{1}{3}$$

19. (b) $\vec{n}_1=3\hat{i}-3\hat{j}+\hat{k}$ and $\vec{n}_2=\hat{i}-\hat{j}+c\hat{k}$

$$\cos 45^\circ=\left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|=\left|\frac{3+3+c}{\sqrt{19}\sqrt{2+c^2}}\right|$$

$$\therefore\ 2(6+c)^2=19(2+c^2)$$
$$\Rightarrow\ 2(36+c^2+12c)=38+19c^2$$
$$\Rightarrow\ 17c^2-24c-34=0$$

20. (c) $L_1=\dfrac{x-7}{-3}=\dfrac{y-6}{2}=\dfrac{z-2}{4}$

and $L_2=\dfrac{x-5}{2}=\dfrac{y-3}{1}=\dfrac{z-4}{3}$

$\therefore$ $C(-3\lambda+7, 2\lambda+6, 4\lambda+2)$
and $D(2\mu+5, \mu+3, 3\mu+4)$

so, $\dfrac{2-3\lambda-2\mu}{2}=\dfrac{3+2\lambda-\mu}{-2}=\dfrac{-2+4\lambda-3\mu}{-1}$

$\therefore\ \lambda=2, \mu=1$
so, $C(1, 10, 10)$ and $D(7, 4, 7)\Rightarrow CD=9$.

21. (d) Equation of plane parallel to L_1 and containing L_2 is
$a(x-5)+b(y-3)+c(z-4)=0$
$\therefore\ 2a+b+3c=0$ and $-3a+2b+4c=0$

$$\Rightarrow\ \frac{a}{-2}=\frac{b}{-17}=\frac{c}{7}$$

So, required plane is $2x+17y-7z=33$.

22. (b) Volume $=[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]=\begin{vmatrix}2 & 3 & -2\\ 6 & -4 & -8\\ 0 & 2 & -5\end{vmatrix}=138.$

DPP 3.4

Single Correct Answer Type

1. (c) $G=(4, 4, 4)$
Let $D=(a, 0, 3-a)$
$GD^2=(a-4)^2+(0-4)^2+(3-a-4)^2$
$=2a^2-6a+33$

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For minimum of GD^2, $a = \frac{3}{2}$

$$GD_{\min} = \sqrt{2\cdot\frac{9}{4} - 6\cdot\frac{3}{2} + 33} = \sqrt{\frac{57}{2}}$$

2. (b) Equation of a plane passing through the line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ is

$3x - y + 2z - 1 + \lambda(x + 2y - z - 2) = 0$

$\Rightarrow (3 + \lambda)x + (-1 + 2\lambda)y + (2 - \lambda)z + (-1 - 2\lambda) = 0$

Since it must be perpendicular to the given plane

$\Rightarrow (3 + \lambda)3 + (-1 + 2\lambda)2 + (2 - \lambda)1 = 0$

$\Rightarrow 6\lambda = -9 \Rightarrow \lambda = -3/2$

$\Rightarrow$ Plane is $3x - 8y + 7z + 4 = 0$

Now line of projection is line of intersection of planes $3x - 8y + 7z + 4 = 0$ and $3x + 2y + z = 0$.

3. (a) Let $O(\alpha, \beta, \gamma)$ be orthocenter.

Now, $AO \perp BC$

$\Rightarrow b\beta = c\gamma$

Similarly, $a\alpha = b\beta = c\gamma = k$

$$\therefore \quad \alpha = \frac{k}{a}, \beta = \frac{k}{b}, \gamma = \frac{k}{c}$$

The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ contains (α, β, γ).

$$\therefore \quad k\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right) = 1$$

4. (a) Any plane passing through first line is $2x + y + z - 1 + \lambda(3x + y + 2z - 2) = 0$.

Line $x = y = z$ is passing through the point $O(0, 0, 0)$.

Required shortest distance

= distance of O from the member plane of above family which is parallel to the line $x = y = z$

If plane is parallel to the line,

$(2x + 3\lambda)1 + (1 + \lambda)1 + (1 + 2\lambda)1 = 0$

$$\Rightarrow \quad \lambda = -\frac{2}{3}$$

Equation of plane is

$3(2x + y + z - 1) - 2(3x + y + 2z - 2) = 0$

or $y - z + 1 = 0$

Its distance from $(0, 0, 0)$ is $\frac{1}{\sqrt{2}}$.

5. (a) Centre and radius of the sphere are $(-1, 1, -1)$ and 3, respectively.

Distance of $(-1, 1, -1)$ from the plane is $\left|\frac{-2 + 3 - 6 + k}{\sqrt{4 + 9 + 36}}\right|$.

Since the plane is tangent to the sphere

$$\therefore \quad \left|\frac{k - 5}{7}\right| = 3$$

$\therefore k = -16, 26$

6. (a) $xy + yz + zx + y^2 = 0$

$\Rightarrow y(x + y) + z(x + y) = 0$

$\Rightarrow$ Planes are $x + y = 0$ and $y + z = 0$

$\Rightarrow$ Line of intersection is $x = -y = z$

Distance of $P(1, 1, 1)$ from $O(0, 0, 0)$ is $\sqrt{3}$.

Projection of OP on line having direction ratios $1, -1, 1$ is $\frac{1}{\sqrt{3}}$.

Then required shortest distance $= \sqrt{3 - \frac{1}{3}} = \sqrt{\frac{8}{3}}$

7. (a) The equation of plane in the intercept form is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Volume of tetrahedron $OABC$ is

$$v = \frac{abc}{6} = 64 \Rightarrow abc = 384$$

Foot of perpendicular from $(0, 0, 0)$ on this plane is

$$\frac{x}{1/a} = \frac{y}{1/b} = \frac{z}{1/c} = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}} = k$$

$$\Rightarrow \quad x = \frac{k}{a}, y = \frac{k}{b}, z = \frac{k}{c}$$

and $\frac{1}{k} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$

$$\Rightarrow \quad \frac{1}{k} = \frac{x^2 + y^2 + z^2}{k^2} \Rightarrow x^2 + y^2 + z^2 = k$$

$\therefore (x^2 + y^2 + z^2)^3 = k^3 = abc\ xyz = 384\ xyz$ is the required locus.

8. (b) Projection of line $\frac{x}{2} = \frac{y - 1}{2} = \frac{z - 1}{1}$ on a plane P is $\frac{x}{1} = \frac{y - 1}{1} = \frac{z - 1}{-1}$.

$\therefore$ Plane through these lines is perpendicular to the plane P.

Normal to the plane determined by the given lines is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -3\hat{i} + 3\hat{j}$$

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$\therefore$ Direction ratios normal to the required plane is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & 0 \\ 1 & 1 & -1 \end{vmatrix} = -3\hat{i} - 3\hat{j} - 6\hat{k}$$

$\therefore$ Equation of the plane is $x + y + 2z - 3 = 0$ as it passes through (0, 1, 1).

Its distance from origin is $\sqrt{\frac{3}{2}}$.

9. (a) For sphere $x^2 + y^2 + z^2 = 9$, (0, 0, 0) is center and radius is 3.

Image of (0, 0, 0) in the plane $2x + 3y + 4z - 29 = 0$ is

$$\frac{x-0}{2} = \frac{y-0}{3} = \frac{z-0}{4} = \frac{-2(0+0+0-29)}{2^2+3^2+4^2}$$

$\therefore$ image is (4, 6, 8)

$\therefore$ Equation of image sphere is

$(x-4)^2 + (y-6)^2 + (z-8)^2 = 9$

or $x^2 + y^2 + z^2 - 8x - 12y - 16z + 107 = 0$

10. (c) Any point on line L: $\frac{x}{1} = \frac{y}{1} = \frac{z}{-1}$ is $Q(r, r, -r)$.

Let point P be (α, β, γ).

$PQ \perp L$

$\therefore$ $(\alpha - r)\cdot 1 + (\beta - r)\cdot 1 + (\gamma + r)(-1) = 0$

$\therefore$ $r = \frac{\alpha + \beta - \gamma}{3}$

$\therefore$ $PQ^2 = \frac{2}{3}\{\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta + \beta\gamma + \alpha\cdot\gamma\}$

But PQ = 2(distance of P from the plane $x + y + z = 0$)

$\therefore$ $\frac{2}{3}(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta + \beta\gamma + \gamma\alpha) = 4\left\{\frac{\alpha + \beta + \gamma}{\sqrt{3}}\right\}^2$

$\therefore$ $\alpha^2 + \beta^2 + \gamma^2 + 5\alpha\beta + 3\beta\gamma + \gamma\alpha = 0$

$\therefore$ Locus is $x^2 + y^2 + z^2 + 5xy + 3yz + zx = 0$

11. (c) Let plane PQR be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Let the shpere be

$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

It passes through the origin.

$\therefore$ $d = 0$

It passes through $P(a, 0, 0)$.

$\therefore$ $a = -2u$

Similarly $b = -2v$ and $c = -2w$.

Also, plane passes through $(\alpha - \beta, \beta - \gamma, \gamma - \alpha)$.

$\therefore$ $\frac{\alpha - \beta}{-2u} + \frac{\beta - \gamma}{-2v} + \frac{\gamma - \alpha}{-2w} = 1$

$\therefore$ Locus of centre $(-u, -v, -w)$ is

$(\alpha - \beta)yz + (\beta - \gamma)zx + (\gamma - \alpha)xy = 2xyz$

12. (a) Given plane is $x + 2y + 3z - 4 = 0$ (1)

Plane through the point $P(1, 1, 1)$ having normal (1, 1, 1) is

$1(x - 1) + 1(y - 1) + 1(z - 1) = 0$

or $x + y + z - 3 = 0$ (2)

Locus of Q is the line of intersection of the planes (1) and (2).

Which is $\frac{x}{1} = \frac{y-5}{-2} = \frac{z+2}{1}$.

Multiple Correct Answers Type

13. (a, c)

Let the equation of plane be $lx + my + nz = 0$, where l, m, n be d.c's.

$\Rightarrow$ $l^2 + m^2 + n^2 = 1$ (1)

Plane is parallel to $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+1}{-2}$

So, $2l - m - 2n = 0$ (2)

Also, $\frac{l - 3m - n}{\sqrt{l^2 + m^2 + n^2}} = \pm\frac{5}{3}$

$\Rightarrow$ $l - 3m - n = \pm\frac{5}{3}$ (3)

Solving (1), (2) and (3), we get equation of plane as $x - 2y + 2z = 0$ or $2x + 2y + z = 0$.

14. (a, b, c)

Treating the three equations as a 3×3 linear system, we have

$$D = \begin{vmatrix} a & 4 & 1 \\ 0 & 2 & 3 \\ 3 & 0 & -b \end{vmatrix} = 30 - 2ab$$

$$D_x = \begin{vmatrix} 0 & 4 & 1 \\ 1 & 2 & 3 \\ -2 & 0 & -b \end{vmatrix} = 4b - 20$$

Similarly, $D_y = -ab + 6a - 3$, $D_z = -4a + 12$

If $ab \neq 15$ the system will have unique solution.

$\Rightarrow$ Planes will meet in a point.

If $ab = 15$, $D_x = \frac{60}{a} - 20$, $D_y = 6a - 18$,

$D_z = -4a + 12$

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If $a = 3$, $D_x = D_y = D_z = 0$

For $a \neq 3$ or $b \neq 5$ at least one of the determinants D_x, D_y and D_z becomes non zero.

$\Rightarrow$ Planes will have no common point.

Thus, all choice are correct.

15. (a, b, c)

Any point $(\lambda, 2\lambda, 3\lambda)$ on line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ intersects the planes $3\beta^2 x + 3(1 - 2\alpha)y + z - 3 = 0$ and $\frac{1}{2}\{6\alpha^2 x + 3(1 - 2\beta)y + 2z\} + 3 = 0$ at same point.

$$\therefore \quad \frac{3}{3\beta^2 + 6(1-2\alpha) + 3} = \frac{-6}{6\alpha^2 + 6(1-2\beta) + 6}$$

$$\Rightarrow \quad 2(\beta - 1)^2 + 3(\alpha - 2)^2 = 0$$

$$\Rightarrow \quad \alpha = 2, \beta = 1$$

Comprehension Type

16. (a) $AB = \sqrt{1+1+4} = \sqrt{6}$

$d_1 = \frac{1}{\sqrt{3}}$, $d_2 = \frac{3}{\sqrt{3}}$ (where d_1 and d_2 are perpendicular distances of A and B from the plane, respectively)

$$\therefore \quad A'B' = \sqrt{(AB)^2 - (d_1 - d_2)^2}$$

$$= \sqrt{(\sqrt{6})^2 - \left(\frac{1}{\sqrt{3}} - \frac{3}{\sqrt{3}}\right)^2}$$

$$= \sqrt{6 - \frac{4}{3}} = \sqrt{\frac{14}{3}}$$

17. (a) Direction ratios of AB are $1, -1, -2$.

Direction ratios of normal to plane are $1, 1, 1$.

$\therefore$ Direction ratios of required line are given by

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = (1, -3, 2)$$

$\therefore$ Required line:

$$\frac{x - \frac{3}{2}}{1} = \frac{y - \frac{1}{2}}{-3} = \frac{z + 2}{2}$$

$$\left(\because \left(\frac{3}{2}, \frac{1}{2}, -2\right) \text{ lies in the plane}\right)$$

18. (d) Equation of family of planes containing line of intersection of planes P_1 and P_2 is given by

$$(x + y + 2z - 3) + \lambda(x - 2y + z - 4) = 0$$

or $\quad (1 + \lambda)x + (1 - 2\lambda)y + (2 + \lambda)z - 3 - 4\lambda = 0 \quad (1)$

Now projection of length of line segement AB is greatest then AB must be parallel to plane (1).

$\therefore$ d.r.'s of AB are $2, -1, 3$

$\therefore$ $2(1 + \lambda) - 1(1 - 2\lambda) + 3(2 + \lambda) = 0$

$\Rightarrow$ $7\lambda + 7 = 0 \quad \Rightarrow \quad \lambda = -1$

$\therefore$ equation of plane is $3y + z + 1 = 0$

19. (c) Projection of length of line segment is least if AB is perpendicular to plane (1).

$$\therefore \quad \frac{1+\lambda}{2} = \frac{1-2\lambda}{-1} = \frac{2+\lambda}{3} \quad \Rightarrow \quad \lambda = 1$$

$\therefore$ Equation of plane P_4 is $2x - y + 3z - 7 = 0$

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